COMPETITIVE STRATEGY FOR OPEN AND USER INNOVATION*

GASTÓN LLANES†

ABSTRACT. I study the incentives to open technologies in imperfectly competitive markets with user innovation. Firms may choose to open part of their knowledge or private information so that it becomes freely accessible to users. Openness decisions are governed by a trade-off between collaboration and appropriability: by becoming more open, a firm encourages user innovation but hampers its ability to capture value. I find that large firms are less open and invest more in product development than small firms, and that firms react to greater openness from rivals by becoming more open. I also show that compatibility and spillovers have a negative effect on openness, and that firms become more open as the number of competitors increases.

KEYWORDS: Competitive Strategy, Open Innovation, User Innovation, Openness Choice, Compatibility Choice, Spillovers, Asymmetric Equilibria, Appropriability, Open-Source Software, Open Standards, Network Effects.

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†Pontificia Universidad Católica de Chile, School of Business, gaston@llanes.com.ar.

1. Introduction

An important question for competitive and technology strategy is whether firms should follow open or closed approaches to product development and intellectual-property management. In this paper, I study the incentives to open technologies in imperfectly competitive markets with user innovation.

Openness enables and facilitates user innovation (von Hippel, 1988, 2005) but may lower a firm's ability to capture value (Arrow, 1962).[1] For example, IBM's creation of an open standard around the PC allowed third-party suppliers of peripheral devices, expansion cards, and software to develop PC-compatible products, but also led to the entry of a myriad of clone manufacturers that eroded IBM's market dominance. Likewise, releasing software under an open-source license allows users to contribute code and provide valuable feedback, but may also cannibalize sales of the firm's proprietary offerings.[2] Thus, openness leads to a *trade-off between collaboration and appropriability*, which affects firms' decisions to open technologies (West, 2003; Henkel, 2006; Henkel, Schöberl, and Alexy, 2014).

In a competitive environment, firms' openness decisions are also affected by strategic considerations. In the mid 2000s, IBM's proprietary product WebSphere Application Server was facing strong competition from the open-source product JBoss Application Server in the middleware market. IBM reacted by buying JBoss's main rival (a developer named Gluecode) and releasing its source code under an open-source license. JBoss's CEO, Marc Fleury, complained at the time that IBM's intention was to "kill JBoss," and warned that Gluecode would hurt sales of IBM's WebSphere as much as JBoss's.[3]

[1]Chesbrough and Appleyard (2007) define openness as the "pooling of knowledge for innovative purposes where the contributors have access to the inputs of others and cannot exert exclusive rights over the resultant innovation." This definition encompasses ideas such as open innovation (Chesbrough, 2003), collective invention (Allen, 1983), and free and open-source software (Raymond, 1999; Stallman, 2002).
[2]Bonaccorsi and Rossi (2004), Shah (2006), Lakhani and Wolf (2005), and Roberts, Hann, and Slaughter (2006) show that user need is one of the main motivations for contributing to open-source projects.
[3]"Open source smack-down," Forbes, June 15, 2005, available at http://www.forbes.com/2005/06/15/jboss-ibm-linux_cz_dl_0615jboss.html.

In this example, IBM responded to competition from a firm that was more open by becoming more open, even though doing so meant creating competition for its own proprietary product. Likewise, Facebook decided to open part of its platform's source code as a response to OpenSocial–an open-source platform used by Google, MySpace, and LinkedIn[4]–and Microsoft became more open after the success of open-source products such as Linux and Chrome.[5]

Extant papers on open and user innovation focus on understanding individual developers' incentives (Johnson, 2002; Polanski, 2007), or study competition between a for-profit firm and a community of non-strategic developers (Casadesus-Masanell and Ghemawat, 2006; Athey and Ellison, 2010; Casadesus-Masanell and Llanes, 2011). In this paper, I contribute to the literature by studying the incentives to open part of a firm's knowledge in a market with imperfectly competitive for-profit firms and user innovators.[6]

From a methodological point of view, the paper links open and user innovation to the literature on network effects. Users provide innovations that benefit other consumers, so the willingness to adopt a firm's product depends on its expected number of users.[7] Given that users can innovate more when they access more knowledge, *the choice of openness determines the intensity of network effects.* This mechanism is absent in previous works studying network effects.

[4]"Facebook open-sources a 'significant part' of its platform," Cnet, June 2, 2008, available at http://cnet.com/news/facebook-open-sources-a-significant-part-of-its-platform/.

[5]While in 2001 Microsoft Windows director James Allchin, stated that "open-source is an intellectual property destroyer [...] I can't imagine something that could be worse than this for the software business," in 2010 Microsoft's interoperability general manager Jean Paoli said "We love open source." See "Dead and buried: Microsoft's holy war on open-source software," Cnet, June 1, 2014, available at https://www.cnet.com/news/dead-and-buried-microsofts-holy-war-on-open-source-software/ and "Microsoft: 'We love open source'," Network World, August 23, 2010, available at https://www.networkworld.com/ article/2216878/windows/microsoft—we-love-open-source-.html.

[6]See also Bessen (2006) and Niedermayer (2013), who show that open source allows firms to overcome organizational and contractual problems, and Reisinger, Ressner, Schmidtke, and Thomes (2014), who study open source as a mechanism to develop a public good with private investments. Llanes and de Elejalde (2013) study a model of competition between for-profit open-source and proprietary firms, but do not allow for partial knowledge disclosures or user innovation.

[7]Oh and Jeon, 2007, present evidence of membership herding in open-source projects, which is consistent with the existence of network effects.

The paper has two main results. First, I show that large firms are less open and invest more in product development than small firms. If users expect a firm to have a larger user base, they expect it to benefit more from user innovation. Thus, the firm can lower its openness degree and increase its price without losing too many customers. Returns to investment, on the other hand, increase with firm size, because large firms enjoy larger product-market revenues (they set higher prices and have a larger market share), and because user investments increase with the number of users and are complementary to firm investments.

This result explains the general observation that firms with a large market share, such as IBM in the middleware market, tend to be less open and invest more in product development than small firms, such as JBoss. Similarly, Microsoft is larger, less open, and has a higher investment than Novell and Red Hat in the server operating systems market; and Nvidia is larger, less open, and has a higher investment than ATI/AMD in the graphics processing units market.

The result is also consistent with the findings of Bonaccorsi, Giannangeli, and Rossi (2006), who show that firm size is negatively correlated with openness. To the best of my knowledge, this paper is the first to provide a formal link between firm size and openness.

Second, I show that firms react to greater openness from rivals by becoming more open. This result is consistent with the examples of IBM, Facebook, and Microsoft mentioned above. In a similar vein, Toyota responded to Tesla's decision to release its electric-car patents by releasing its patents on the competing fuel-cell technology, and Microsoft opened up Azure to respond to competition from Amazon Web Services, which was more open.

I also show that spillovers have a negative effect on openness, and that firms become more open as the number of competitors increases. Spillovers imply that the knowledge a firm discloses can be accessed by its rivals. As the intensity of spillovers increases, firms become less open and equilibrium profits increase. Therefore, firms may benefit from coordinating on a high level of spillovers (for example, by making their products more compatible) to induce an equilibrium with a smaller openness degree. The result that firms react to entry by competitors by becoming more open explains Apple's decision to open Swift after the entry of cross-platform solutions for developing smartphone applications, such as Microsoft's Xamarin and Adobe's Cordova.

The main contribution of this paper is to clarify the role of strategic and competitive factors on openness decisions. Its results have direct managerial implications. In particular, the paper shows the best openness and investment strategies for small and large firms, and explains how to respond to increases in openness by rival firms. In section 10, I discuss real-world examples to illustrate the paper's findings. In the conclusion, I discuss the limitations of the model and present ideas for further research.

2. The model

Two firms, $i = 1, 2$, develop and sell products to a continuum of users with unit mass. User demands depend on prices and knowledge investments. Firms can protect their knowledge with intellectual property rights or secrecy, but they can also choose to disclose (open) part of their knowledge so that it becomes freely accessible to users.

Users can use the firm's disclosed information to modify and improve the firm's products through *user innovation*, which benefits the firm as these innovations become available to other users. However, users can also use the disclosed information to build their own version of the firm's product, which limits the price the firm can charge for the product that uses its proprietary (undisclosed) knowledge.

Thus, when choosing its openness degree, a firm faces a *trade-off between collaboration and appropriability*: greater openness benefits the firm because other agents contribute to increasing the firm's knowledge, but it also reduces the firm's ability to capture value.

The model describes the fundamental trade-off at play in a variety of cases. In the case of software, a firm that releases part of its source code under an open-source license allows for greater user innovation, but at the same time creates competition for the product that includes all of its source code. Oracle's MySQL, for example, distributes an open-source community edition, which competes against its paid enterprise edition.

Likewise, a firm contributing its patents to an open standard benefits when standards' adopters provide feedback and develop complementary products, but may have to commit to license its patents on FRAND terms (fair, reasonable, and non-discriminatory), which sets a cap on the royalty fees it can charge (see Simcoe, 2006, for a discussion of the trade-off between openness and appropriability in standard setting). An example is IBM's creation of an open standard around the PC, which allowed for greater adoption and development of complementary products, but decreased IBM's appropriability after a multitude of PC-compatible manufacturers entered the market.

The quality of the *proprietary good* of firm i (the product that includes the firm's open and proprietary knowledge) is $q_i^p = x_i + z_i$, where x_i represents firm i's stock of knowledge and z_i represents the innovations contributed by firm i's users. The quality of the product based on firm i's open knowledge is $q_i^o = \phi_i x_i + z_i$, where ϕ_i is the fraction of knowledge disclosed by the firm (openness degree). I will refer to this product as firm i's *open product*.

In the case of a software program, for example, x_i is the number of lines of the source code of the firm, ϕ_i is the fraction of source code released under the open-source license, and z_i is the number of lines of code contributed by user/developers.

Let $\sigma \in [0, 1]$ be the intensity of user innovation (contributions per user), and let s_i^e be the expected measure of users of the firm's products. User innovation is $z_i = \sigma \phi_i x_i s_i^e$. This simple functional form captures two effects: (i) users can innovate more if they can access more knowledge (intensive margin), and (ii) more users imply more user innovation (extensive margin). I am implicitly assuming that individual user investments are exogenously given (each user innovates a fraction σ on top of the knowledge she can access, and total user innovation is the sum of the innovations of each user). In section 9 I extend the model to allow for endogenous user investments.

Following Katz and Shapiro (1985), I assume players take the expected size of the network (s_i^e) as given when making their decisions, and that such expectations are fulfilled in equilibrium ($s_i^e = s_i^*$).

Each user chooses a product to maximize her indirect utility. User k's indirect utility for consuming firm i's proprietary product is

$$v_{ik}^p = q_i^p - p_i + \varepsilon_{ik}, \tag{1}$$

and her indirect utility for consuming firm i's open product is

$$v_{ik}^o = q_i^o + \varepsilon_{ik}, \tag{2}$$

where p_i is the price of firm i's proprietary product and ε_{ik} is an idiosyncratic taste parameter. Let s_i^p and s_i^o be the share of users choosing the proprietary and open products of firm i, and let $s_i = s_i^p + s_i^o$ be the total share of users of firm i. For simplicity, assume users choose the proprietary good when they are indifferent between the open and proprietary products of a firm.

The taste differential $\omega_k = \varepsilon_{1k} - \varepsilon_{2k}$ has a cumulative density function F. The probability density function, $f = F'$, is symmetric, strictly positive in the support $(-\infty, \infty)$, and centered around zero, with $f'(\omega) > 0$ for $\omega < 0$, $f'(0) = 0$, and $f'(\omega) < 0$ for $\omega > 0$. The hazard function, $h = f/(1 - F)$, is increasing. Let F^{-1} be the inverse function of the cumulative density function. Symmetry implies $f(\omega) = f(-\omega)$ and $F(\omega) = 1 - F(-\omega)$.

The products of different firms are differentiated vertically and horizontally, but the open and proprietary goods of the same firm are differentiated only along the vertical dimension: given that they are based on the same basic knowledge, they share horizontal characteristics, and the only difference between the two goods is that the proprietary product incorporates more knowledge than the open product.[8]

[8] An interesting direction for future research is to study horizontal differentiation of a firm's open and proprietary products, in which case firms may use their product portfolio to segment the market and discriminate among users with different needs and propensity to innovate.

The model is a two-stage non-cooperative game. In the first stage, firms choose ϕ_i and p_i simultaneously to maximize profits $\pi_i = p_i\, s_i^p$, taking expectations s_i^e as given. In the second stage, users observe firms' prices and qualities, and choose their preferred variety. In equilibrium, expectations are fulfilled ($s_i^e = s_i^*$). The equilibrium concept is subgame-perfect Nash equilibrium with fulfilled expectations.

3. Equilibria

Substituting q_i^p and q_i^o into (1) and (2), I obtain

$$\begin{aligned} v_{ik}^p &= x_i + \sigma\,\phi_i\,x_i\,s_i^e - p_i + \varepsilon_{ik}, \qquad (3)\\ v_{ik}^o &= \phi_i\,x_i + \sigma\,\phi_i\,x_i\,s_i^e + \varepsilon_{ik}. \end{aligned}$$

Comparing these expressions, it is straightforward to see that $s_i^p = 0$ if $\phi_i\,x_i > x_i - p_i$ and $s_i^o = 0$ if $\phi_i\,x_i \le x_i - p_i$. In equilibrium, it must hold that $\phi_i\,x_i \le x_i - p_i$. Otherwise, users would prefer firm i's open product to its proprietary product, and firm i would have zero profits. Also, if $\phi_i\,x_i < x_i - p_i$ firm i can raise ϕ_i without changing p_i, which increases user innovation and the demand for its proprietary good, while keeping $s_i^o = 0$. In equilibrium, it must hold that $\phi_i^*\,x_i = x_i - p_i^*$, which is equivalent to $p_i^* = (1-\phi_i^*)\,x_i$.

Taking this result into account, in what follows, I let $p_i = (1-\phi_i)\,x_i$ and focus on the choice of openness degree. The model captures in a simple way one of the fundamental trade-offs firms face when choosing their openness level: by increasing ϕ_i, a firm encourages user innovation, but it also raises competition between its open and proprietary offerings, thus lowering the price it can charge for its proprietary product.

Users observe qualities and prices and choose their preferred product. The demand of firm i's proprietary product is

$$\begin{aligned} s^d(\phi_i,\phi_j,x_i,x_j,s_i^e) &= \Pr\big((1+\sigma s_i^e)\,\phi_i\,x_i + \varepsilon_{ik} \ge (1+\sigma(1-s_j^e))\,\phi_j\,x_j + \varepsilon_{jk}\big),\\ &= F\big((1+\sigma s_i^e)\,\phi_i\,x_i - (1+\sigma(1-s_j^e))\,\phi_j\,x_j\big), \qquad (4) \end{aligned}$$

where j is firm i's rival. In the first stage, firm i chooses $\phi_i \in [0,1]$ to maximize

$$\pi_i(\phi_i, \phi_j, x_i, x_j, s_i^e) = (1-\phi_i)\; x_i\;\; s^d(\phi_i, \phi_j, x_i, x_j, s_i^e),$$

taking s_i^e as given. Assume the maximization problem has an internal solution and let $y_i = (1+\sigma s_i^e)\,\phi_i\,x_i - (1+\sigma(1-s_j^e))\,\phi_j\,x_j$ denote firm i's marginal user. The first-order condition is $-x_i\,F(y_i) + (1-\phi_i)\,x_i\,f(y_i)\,(1+\sigma s_i^e)\,x_i = 0$, and the increasing hazard ratio property guarantees that the second-order condition holds. From the first-order condition, I obtain

$$\frac{f(y_i)\,(1-\phi_i)\,x_i}{F(y_i)} = \frac{1}{1+\sigma\, s_i^e}. \tag{5}$$

Given that price is equal to $(1-\phi_i)\,x_i$, equation (5) relates the price elasticity of demand to user expectations and the intensity of user innovation. If $\sigma = 0$, we obtain the standard result from microeconomics that an imperfectly competitive firm with zero marginal costs would choose a price such that the price elasticity of demand equals 1. If $\sigma > 0$, on the other hand, the firm chooses a price in the inelastic part of the demand curve.

The intuition behind this result is that user innovation is larger the more open the firm is, and openness is negatively related to price in equilibrium. Thus, the firm finds it optimal to choose a lower price than it would choose if user innovation were absent. User innovation acts as an ad valorem tax (the factor $1+\sigma\, s_i^e$ magnifies the effect of a change in price for users), which lowers the firm's optimal price. Given that elasticity increases with price (in absolute value), a lower price means that the firm ends up pricing in the inelastic part of the demand function.[9]

In what follows, I focus on the case of symmetric investments in product quality ($x_1 = x_2 = x$), and postpone the discussion of asymmetric investments until section 7.

[9] The increasing hazard ratio and symmetry of the pdf imply that f/F is decreasing. Given that y_i increases with ϕ_i (decreases with p_i), the elasticity $f(y_i)\,(1-\phi_i)\,x/F(y_i)$ decreases with ϕ_i (increases with p_i).

In equilibrium, expectations are fulfilled ($s_i^e = s_i^*$). Working with the first-order condition, I obtain the following expression for the marginal user in equilibrium:

$$y_i^* \quad = \quad (1 - 2\,F(y_i^*)) \left(\frac{1}{f(y_i^*)} - \sigma\, x \right). \tag{6}$$

This expression can be restated in terms of market shares,

$$s_i^* \quad = \quad F\left((1 - 2\, s_i^*) \left(\frac{1}{f(F^{-1}(s_i^*))} - \sigma\, x \right) \right), \tag{7}$$

and from (5), it follows that the equilibrium openness degree is

$$\phi_i^* \quad = \quad 1 - \frac{s_i^*}{(1 + \sigma s_i^*)\, x\, f(y_i^*)}. \tag{8}$$

Proposition 1 characterizes the symmetric equilibrium, and Proposition 2 characterizes asymmetric equilibria. All proofs are in Appendix A.

Proposition 1 (Symmetric equilibrium)**.** *A symmetric equilibrium (*$s_1^* = s_2^* = 1/2$*) exists and is unique. In the symmetric equilibrium, firms become more open as (i) the intensity of user innovation* σ *becomes larger, (ii) the knowledge stock* x *becomes larger, (iii) and users' sensitivity to changes in quality and prices* $f(0)$ *increases.*

To understand why firms become more open as σ and x increase, note that if user innovation or the knowledge stock is larger, users can introduce more innovations for a given openness level. An increase in σ or x increases the sensitivity of user innovation and profits to changes in the openness degree, which causes firms to become more open.

Proposition 1 also shows that openness increases with demand sensitivity. To understand this result, note that when demand becomes more sensitive to changes in quality and prices, the demand-stealing effect of encouraging more user innovation increases, and thus firms' compete more fiercely for users. As a result, firms end up being more open than what they would be if they were isolated from competition.

Proposition 2 (Asymmetric equilibria)**.** *Asymmetric equilibria exist if and only if* $\sigma x f(0) > 3/2$. *In an asymmetric equilibrium, the firm with larger market share is less open, sets higher prices, and has higher profits than its rival.*

Ex-post asymmetries arise even if firms are ex-ante symmetric in terms of product characteristics, and are caused by the interdependence of users' adoption decisions: if users expect the product of one firm to be adopted more, they expect it to benefit more from user innovation and will be more inclined to buy it over the rival's firm product, which in turn justifies users' more favorable expectations.

Proposition 2 shows that larger firms (in terms of market share) are less open than smaller firms. Large firms benefit more from user innovation for a given openness degree. Thus, the firm can lower its openness degree and increase its price without losing too many customers.

I postpone the analysis of comparative statics in asymmetric equilibria until section 4. Proposition 3 further characterizes strategic interaction.

Proposition 3 (Strategic complementarity and stability)**.** *Openness degrees are strategic complements. All equilibria (symmetric and asymmetric) are locally stable.*

Proposition 3 shows that openness degrees, ϕ_i, are strategic complements (Bulow, Geanakoplos, and Klemperer, 1985). Thus, an increase in the openness degree of one firm leads to an increase in the openness degree of its rival.

Proposition 3 also shows that all equilibria are stable. Intuitively, if there is a small perturbation in players' actions near an equilibrium, the model will return to this equilibrium if players play reactive strategies according to their best-response functions. If an equilibrium were unstable, a small change in the action of one firm would trigger a large departure from equilibrium play, which implies that such an equilibrium would be hard to justify as a prediction of the outcome of the game.

4. Logistic distribution

To illustrate the results of the previous section, I present an example based on the logistic distribution. The cumulative distribution function is

$$F(\omega) = \frac{1}{1 + \exp(-\omega/\mu)},$$

the density function is $f = F(1-F)/\mu$, and the variance is $\pi^2 \mu^2/3$. The larger μ is, the larger the dispersion in user tastes (i.e., the larger the degree of horizontal differentiation).

In the proof of Proposition 2, I defined function $H(s)$ as

$$H(s) \quad = \quad F\left((1-2\,s)\left(\frac{1}{f(F^{-1}(s))} - \sigma\, x\right)\right) - s.$$

Function $H(s)$ can be interpreted as an equilibrium excess demand function given expectations s. To see this, note that $F\left((1-2\,s)\left(\frac{1}{f(F^{-1}(s))} - \sigma\, x\right)\right)$ gives the equilibrium demand consistent with expectations s. If users expect firm i to have a market share of s_i and $F\left((1-2\,s_i)\left(\frac{1}{f(F^{-1}(s_i))} - \sigma\, x\right)\right) > s_i$, demand exceeds expectations. If expectations are fulfilled, there is no excess demand and $H(s) = 0$.

In the case of the logistic distribution, $H(s)$ is given by

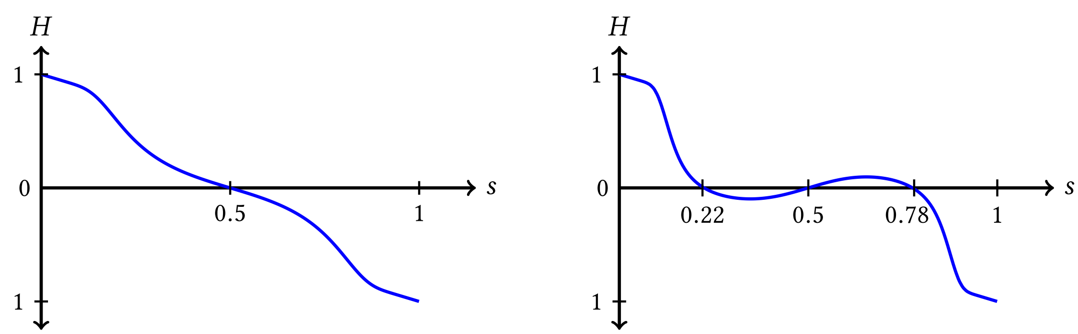
$$H(s) = \left[1 + \exp\left(-\frac{1}{\mu}(1-2s)\left(\frac{\mu}{s(1-s)} - \sigma\, x\right)\right)\right]^{-1} - s.$$

It is straightforward to see that $f(0) = (4\mu)^{-1}$. From Proposition 2, it follows that the number of equilibria depends on $\sigma\, x/\mu$, which compares the importance of user innovation and the knowledge stock to the degree of horizontal differentiation. If $\sigma\, x/\mu < 6$, the unique equilibrium is symmetric. If $\sigma\, x/\mu \geq 6$, there exist three equilibria: the symmetric equilibrium and two asymmetric equilibria in which one of the firms has a larger market share. The openness degree in the symmetric equilibrium is

$$\phi^* = 1 - \frac{4\mu}{(2+\sigma)\, x},$$

which increases with σ and x and decreases with μ.

Figure 1a shows the graph of $H(s)$ for $\sigma = 0.2$, $x = 20$, and $\mu = 1$. In this example, $\sigma x/\mu = 4$, so there is only one equilibrium. Openness degree and profits are $\phi^* = 0.91$ and $\pi^* = 0.91$. Figure 1b shows $H(s)$ for $\sigma = 0.2$, $x = 40$, and $\mu = 1$. In this example, $\sigma x/\mu = 8$, so there are three equilibria: $\{s_1^* = 0.22, s_2^* = 0.78\}$, $\{s_1^* = 0.5, s_2^* = 0.5\}$, and $\{s_1^* = 0.78, s_2^* = 0.22\}$. As for the openness degree and profits, $\phi_i^* = 0.97$ and $\pi_i^* = 0.28$ for $s_i^* = 0.22$, $\phi_i^* = 0.95$ and $\pi_i^* = 0.91$ for $s_i^* = 0.5$, and $\phi_i^* = 0.9$ and $\pi_i^* = 2.99$ for $s_i^* = 0.78$. Therefore, in the asymmetric equilibrium, the small firm is more open and has lower profits than in the symmetric equilibrium, and the large firm is less open and has higher profits than in the symmetric equilibrium.



(A) Unique equilibrium

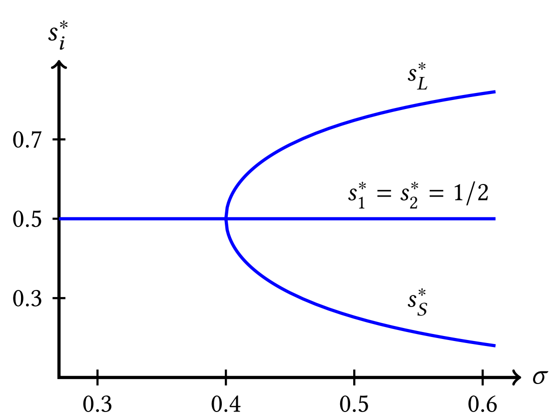
(B) Multiple equilibria

Figure 1. Logistic distribution

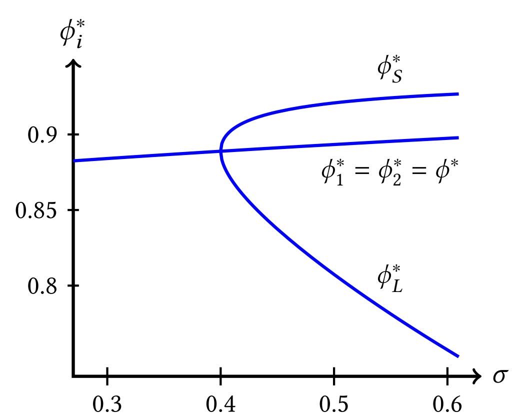
Figure 2 shows the effects of changes in σ on s_i^* and ϕ_i^* for $\mu = 1$ and $x = 15$. Figure 2a shows equilibrium market shares. The middle curve shows the symmetric equilibrium market shares ($s_1^* = s_2^* = 1/2$), and the curve labeled s_L^* (s_S^*) shows the market share of the large (small) firm in an asymmetric equilibrium. For $\sigma \leq \frac{6\mu}{x} = 0.4$, the unique equilibrium is symmetric. For $\sigma > \frac{6\mu}{x} = 0.4$, there exist three equilibria. As σ increases, the asymmetry between the large and the small firm in an asymmetric equilibrium increases: as the intensity of user innovation increases, the effect of expectations on adoption decisions increases, which allows for a larger difference in the market shares consistent with fulfilled expectations.

Figure 2b shows equilibrium openness. The middle curve shows the openness degree in the symmetric equilibrium, and the curve labeled ϕ_L^* (ϕ_S^*) shows the openness degree of the large (small) firm in an asymmetric equilibrium. Consistent with Proposition 2, the large firm is less open than the small firm.

From Proposition 1, we know that openness increases with σ in the symmetric equilibrium. Figure 2b shows that this result continues to hold for the small firm in an asymmetric equilibrium, but not for the large firm. An increase in σ has two effects: (i) it increases the incentives to open technologies holding market shares constant, and (ii) it increases (decreases) the market share of the large (small) firm, which decreases (increases) the incentives to open technologies. In a symmetric equilibrium, the second effect is not present because market shares do not change with σ. In an asymmetric equilibrium, the second effect reinforces the first in the case of the small firm, but goes against the first effect in the case of the large firm. Therefore, a large firm may become more closed following an increase in the intensity of user innovation.



(A) Market share



(B) Openness degree

Figure 2. Comparative statics

5. Social welfare

This section examines how social welfare varies across symmetric and asymmetric equilibria. To obtain a welfare function, I assume the idiosyncratic taste shocks ε_{ik} are independently and identically distributed according to a double exponential distribution. The cumulative distribution is

$$F(\varepsilon_{ik}) = \exp(-\exp(-\nu - \varepsilon_{ik}/\mu)),$$

where ν is Euler's constant ($\nu \approx 0.5772$) and $\mu > 0$ is a constant measuring users' taste heterogeneity. The double exponential distribution implies that the difference $\omega = \varepsilon_{1k} - \varepsilon_{2k}$ is distributed according to a logistic distribution with parameter μ, as in section 4.

Anderson, de Palma, and Thisse (1992, pp. 76–79) show that the discrete-choice model described above has a representative-consumer representation. Let s_i be the quantity of good i by the representative consumer, let I be her income, and let p_i be the price of good i. The utility of the representative consumer is

$$U = \sum q_i^p \, s_i - \mu \sum s_i \log(s_i) + I - \sum p_i \, s_i. \tag{9}$$

This utility embodies two different effects. The first term represents the direct effect from consumption of the different goods (recall that q_i^p represents the quality of the proprietary good of firm i). The second term introduces an entropy effect, which expresses the representative consumer's preference for variety.

The utility function is quasilinear, which implies transferable utility. Social welfare is the sum of consumer utility and firms' profits:

$$\begin{aligned} W &= \sum q_i^p \, s_i - \mu \sum s_i \log(s_i) + I - \sum p_i \, s_i + \sum p_i \, s_i \\ &= \sum (1 + \sigma \phi_i s_i^e) \, x \, s_i - \mu \sum s_i \log(s_i) + I. \end{aligned}$$

Users' taste heterogeneity implies that the second term in W is larger with symmetry. The first term of W could, in principle, be larger under symmetry or asymmetry: network effects are maximized under asymmetry (in an asymmetric equilibrium, more users choose the firm with larger network effects), but the large firm is less open in an asymmetric equilibrium. Proposition 4 shows that for small values of σ, network effects dominate the other factors, and thus social welfare is maximized in asymmetric equilibria.

Proposition 4 (Welfare comparison)**.** *There exists $\hat{\sigma} > 6\,\mu/x$ such that for $\sigma < \hat{\sigma}$ social welfare is larger in asymmetric equilibria than in the symmetric equilibrium.*

Figure 3 shows social welfare for different equilibria as a function of σ for $\mu = 1$ and $x = 15$. The figure shows that the result in Proposition 4 extends to higher values of σ. Thus, asymmetric equilibria provide larger welfare than the symmetric equilibrium. Given that the love-for-variety term in the welfare function is always larger in a symmetric equilibrium, a corollary of Proposition 4 is that the average quality of the goods consumed by users is larger in asymmetric equilibria.

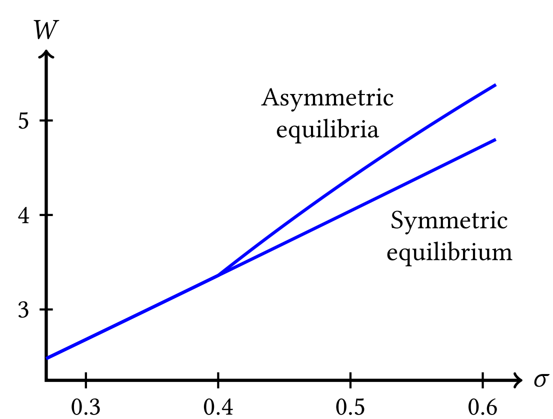


Figure 3. Welfare comparison

6. Partial compatibility and spillovers

In this section, I study compatibility choice and spillovers. Spillovers imply that, by opening its knowledge, a firm exposes itself to the risk that competitors copy its goods and pose a more competitive threat. In the software industry, competing firms may use similar programming languages and architectures, which may allow them to use part of their rival's open-source code. In other industries, firms may share clients with competitors, in which case increasing the flow of information to users may imply larger spillovers to competitors.

Spillovers are embedded in the design philosophy of open source. In one of the most influential writings on open source, Raymond (2000) wrote that "Good programmers know what to write. Great ones know what to rewrite (and reuse)," and that "it is absolutely critical that the coordinator be able to recognize good design ideas from others."

Fershtman and Gandal (2011) study knowledge spillovers in open-source projects. In their database, 63,658 out of 114,751 projects share developers with at least one other project. Fershtman and Gandal show that the success of an open-source project is positively correlated with the project's degree (the number of other projects with which the project shares common developers) and closeness centrality (a measure of how close the project is from other projects in the open-source network), and thus provide evidence of direct and indirect knowledge spillovers.

I assume firms may use a fraction β of the knowledge disclosed by their rivals. The quality of the proprietary good of firm i is $q_i^p = x + \beta\, \phi_j\, x + z_i$, and the quality of the open good of firm i is $q_i^o = \phi_i\, x + \beta\, \phi_j\, x + z_i$, where x is firm i's knowledge, z_i is user innovation, and ϕ_i is the openness degree of firm i. User innovation is given by $z_i = \sigma\, \phi_i\, x\, s_i^e + \sigma\, \beta\, \phi_j\, x\, s_j^e$. Therefore, β measures the extent of spillovers between the products of the two firms. Proposition 5 shows the equilibrium of the game.

Proposition 5 (Partial compatibility and spillovers)**.** *A fulfilled-expectations equilibrium exists. In the unique symmetric equilibrium, firms' profits are increasing in β, openness degrees are decreasing in β, and there exists $\hat{\beta} > 0$ such that the quality of open goods and user surplus are increasing in β if and only if $\beta < \hat{\beta}$.*

Proposition 5 shows that compatibility and openness decisions are substitutes. If a firm decides to participate in an environment with higher spillovers (e.g., by making its product more compatible with other products or by locating closer to competitors), it will optimally choose a smaller openness degree. Larger spillovers make firms less willing to open their technologies. As a result, firms compete less fiercely to capture users and profits increase.

A corollary of this result is that firms may benefit from coordinating on a high level of spillovers, which serves as a commitment device for having a smaller openness degree.

Proposition 5 also shows that the quality of open goods and users' utility may be larger in an environment with more spillovers, even though firms are more closed. If spillovers are small, the beneficial effect of sharing more knowledge among competitors is larger than the negative effect of a smaller openness degree, and the total knowledge embedded in the open good increases with the size of spillovers. This result is consistent with Fershtman and Gandal (2011), who show that the probability of success of an open-source project increases with the number of direct and indirect connections with other projects.

7. Endogenous firms' investments

In this section, I endogenize firms' investments and study how incentives to invest in product quality and openness decisions are related to equilibrium market shares.

I consider the following three-stage non-cooperative game. First, firms choose quality investments x_i. Second, firms choose openness degrees. Third, users observe prices and product characteristics and choose which product to consume. As in section 3, players take expectations as given when making decisions, and expectations are fulfilled in equilibrium. The cost of investment in quality is $c\,x_i^2/2$, where parameter $c > 0$ is large enough for second-order conditions to be satisfied.

Demand is given by (4). Following the same steps as in section 3, it is straightforward to show that given expectations s_i^e, s_j^e and investments x_i, x_j, the second-stage openness degree and market share of firm i solve

$$\hat{\phi}_i = 1 - \frac{\hat{s}_i}{(1+\sigma s_i^e)\, x_i\, f(\hat{y}_i)},$$
$$\hat{s}_i = F\left((1-2\hat{s}_i)\,\frac{1}{f(\hat{y}_i)} + (1+\sigma s_i^e)\, x_i - (1+\sigma s_j^e)\, x_j\right),$$

where $\hat{y}_i$ is firm i's marginal consumer. In the first stage, firm i chooses x_i to maximize $(1-\hat{\phi}_i)\, x_i\, s_i^d$. In the proof of Proposition 6, I show that equilibrium investment is

$$x_i^* = \frac{s_i^* \left(2 - s_i^* \frac{f'(y_i^*)}{f(y_i^*)^2}\right)}{c\left(3 - (1-2s_i^*)\frac{f'(y_i^*)}{f(y_i^*)^2}\right)},$$

where s_i^* is the subgame-perfect equilibrium market share and $y_i^* = F^{-1}(s_i^*)$. Proposition 6 characterizes the equilibrium of the game.

Proposition 6 (Endogenous firms' investments)**.** *An equilibrium exists. In an asymmetric equilibrium, the firm with larger market share invests more, sets a higher price, and has higher profits than its rival.*

Proposition 6 shows that larger firms invest more in product development. Two reasons explain this result. First, larger firms have larger product-market revenues because they set higher prices and have a larger market share than small firms. Thus, they can appropriate a larger share of their investments. Second, user investments increase with the number of users and are complementary to firm investments. Thus, the returns to investment increase with the number of users.

8. Multi-firm competition and entry

In this section, I extend the basic model to introduce competition between $n \geq 2$ firms. To obtain closed-form solutions for demands, I assume the idiosyncratic taste shocks ε_{ik} are distributed according to a double exponential distribution (as in section 5), which leads to logit demands (Anderson, de Palma, and Thisse, 1992). Assuming $x_i = x$ for all i, the demand for product i is

$$s^d(\phi_i, \phi_{-i}, x_i, \mathbf{x}_{-i}, s_i^e, \mathbf{s}_{-i}^e) = \frac{\exp((1 + \sigma\, s_i^e)\, \phi_i\, x/\mu)}{\sum_{j=1}^n \exp((1 + \sigma\, s_j^e)\, \phi_j\, x/\mu)},$$

where the $-i$ subindex represents firms other than i (e.g., ϕ_{-i} is a vector containing the openness degrees of all firms except i). The welfare function is given by (9).

Proposition 7 characterizes the symmetric equilibrium.

Proposition 7 (Multi-firm competition)**.** *There exists a unique symmetric equilibrium ($s_i^* = 1/n$). In this equilibrium, the openness degree is*

$$\phi^* = 1 - \frac{\mu\, n^2}{(n + \sigma)\, x\, (n - 1)},$$

which is increasing in n. Equilibrium prices and profits are decreasing in n.

Proposition 7 shows that firms will react to entry by competitors by becoming more open. An example is Apple's decision to open Swift after the entry of cross-platform solutions for developing smartphone applications, such as Microsoft's Xamarin and Adobe's Cordova.[10]

9. Endogenous user investments

In this section, I extend the model to allow for endogenous user investments. I assume users receive a personal benefit from their investments in R&D, which may be related to own personal gratification or with the profits from the sale of a complementary good or service. I show that the main results of the paper continue to hold with endogenous investments, but I also obtain new results for the effects of an increase in users' individual benefits on total user innovation.

[10]"New cross platform app development tools in 2016," Forbes, April 18, 2016, available at http://forbes.com/sites/tomaslaurinavicius/2016/04/18/new-cross-platform-app-development-tools-in-2016/.

For this section, I return to the duopoly framework of section 2 and assume $x_1 = x_2 = x$. User k's indirect utility for consuming firm i's proprietary product is

$$v^p_{ik} \quad = \quad q^p_i - p_i + \alpha \left(\sigma\, \phi_i\, x\, b_k - \frac{1}{2}\, {b_k}^2\right) + \varepsilon_{ik},$$

where b_k is user k's investment, $\sigma\, \phi_i\, x\, b_k$ is user k's personal benefit from investment, ${b_k}^2/2$ is user k's investment cost, and $\alpha > 0$ is a parameter measuring the importance of individual investments for consumers. User k's indirect utility for consuming firm i's open product is

$$v^o_{ik} \quad = \quad q^o_i + \alpha \left(\sigma\, \phi_i\, x\, b_k - \frac{1}{2}\, {b_k}^2\right) + \varepsilon_{ik}.$$

Total user innovation on the products of firm i is $z_i = \int_{K_i} b_k\, dk$, where K_i is the set of users choosing firm i' products. In equilibrium, firms set $p_i = (1 - \phi_i)\, x$. If user k chooses a product of firm i, her optimal investment is

$$b^*_k = \sigma\, \phi_i\, x.$$

which means that users expect the total investment in user innovation to be $z^*_i = \sigma\, \phi_i\, x\, s^e_i$. In equilibrium, users consume a proprietary good. The utility of choosing the proprietary product of firm i is

$$v^p_{ik} \quad = \quad (1 + \sigma s^e_i)\, \phi_i\, x_i + \alpha\, \frac{1}{2}\, (\sigma\, \phi_i\, x)^2 + \varepsilon_{ik},$$

and firm i's demand is

$$s^d(\phi_i, \phi_j, s^e_i) = F\left((1 + \sigma s^e_i)\, \phi_i\, x + \frac{\alpha}{2}\, (\sigma\, \phi_i\, x)^2 - (1 + \sigma(1 - s^e_i))\, \phi_j\, x - \frac{\alpha}{2}\, \left(\sigma\, \phi_j\, x\right)^2\right).$$

By continuity, previous results continue to hold for α close to 0. The following proposition describes comparative statics as α grows.

Proposition 8 (Endogenous user innovation)**.** *A symmetric equilibrium exists. In this equilibrium, openness degrees and individual user investments increase, and prices and profits decrease, as individual investments become more important (α increases).*

Proposition 8 shows that previous results continue to hold when users' investments are endogenous. It also shows that as individual investments become more important, users become more concerned about the knowledge they can access, and firms compete more aggressively to attract users. As a result, prices and firms' profits decrease.

10. Managerial implications and examples

In this section, I discuss real-world examples to illustrate the managerial implications of the paper.

An example of the result in Proposition 1 that firms become more open as the intensity of user innovation (σ) and the size of the knowledge stock (x) becomes larger is JetBrains' development of IntelliJ IDEA. IntelliJ is a Java integrated development environment (IDE) for developing computer software which supports Java, Perl, Kotlin, HTML, Javascript, PHP, SQL, Python, and Ruby, among other languages. JetBrains offers a community (open source) edition of IntelliJ, and also an ultimate (proprietary) edition, which gives additional functionality to users.[11]

In May 2013, Google announced Android Studio, an IDE for developing Android apps that is based on IntelliJ.[12] Android Studio is a key component of Google's Android strategy. In June 2015, Google ended support for Eclipse (the previous recommended IDE for Android apps) and suggested developers switch to Android Studio.[13]

Google's adoption of IntelliJ implied an increase in the intensity of user innovation. Figure 4 shows the evolution of IntelliJ's openness degree (calculated as the file size of the

[11]See http://jetbrains.com/idea/features/editions_comparison_matrix.html.
[12]"Google eases Android app development with a new IDE," PC World, May 16, 2013, available at http://pcworld.com/article/2038916/.
[13]See http://android-developers.googleblog.com/2015/06/an-update-on-eclipse-android-developer.html.

community edition divided over the file size of the ultimate edition) between 2012 (before Google's announcement) and 2017. Version numbers are indicated next to data points. Data and sources are discussed in Appendix B.

Figure 4 shows that IntelliJ's openness degree has been steadily increasing since its adoption by Google. Given that the size of the ultimate edition (which includes all knowledge) has also been increasing between 2012 and 2017, Figure 4 is consistent with the result that increases in σ and x increase the openness degree, which provides partial support to the findings of Proposition 1.

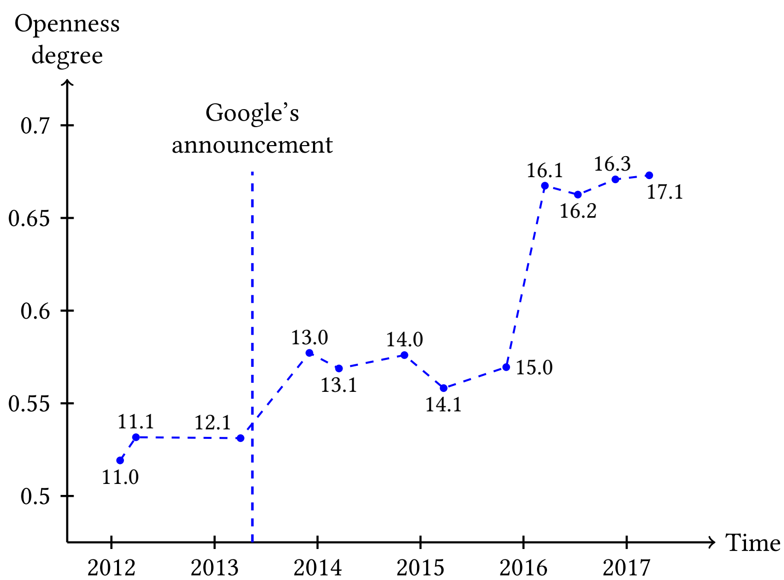


FIGURE 4. Evolution of IntelliJ's openness degree

The result in Proposition 2 that larger firms are less open than smaller firms helps explain why large firms, such as IBM in the middleware market, tend to be more closed than small firms, such as JBoss, as in the example discussed in the introduction. Likewise, in the desktop and server operating system markets, Windows is less open and has a larger market share than Linux's contributors, such as Novell and Red Hat. Another example is the graphics processing units market, in which Nvidia has a larger market share and is less open than its main rival ATI/AMD.[14]

The results of Proposition 2 are also consistent with the observations of recent surveys. Seppä (2006) finds open-source firms tend to be smaller than proprietary firms, Bonaccorsi and Rossi (2004) show the most important motive for firms to participate in open-source projects is that doing so allows small firms to innovate, and Bonaccorsi, Giannangeli, and Rossi (2006), show that firm size is negatively correlated with openness.

The strategic-complementarity result in Proposition 3 explains why firms respond to increased openness by rivals by becoming more open, as in the cases of IBM, Facebook, and Microsoft described in the introduction.[15] Other examples are Toyota's decision to release its patents on the competing fuel-cell technology as an answer to Tesla's decision to release its electric-car patents, and Microsoft's decision to open up Azure as a response to competition from the more open Amazon Web Services.[16]

The result in Proposition 6 is related to the observation that larger firms tend to invest more in R&D than small firms. For example, IBM invests more than JBoss, Microsoft invests more than Novell and Red Hat, and Nvidia invests more than ATI/AMD. An interesting direction for further research would be to test whether this result holds more generally in the data and other industries.

[14]Nvidia has a market share of over 70% in the GPU market (see https://jonpeddie.com/ press-releases/details/add-in-board-market-decreased-in-q117-from-last-quarter-with-nvidia-gaining). Nvidia does not support any open source drivers. ATI/AMD, in contrast, contributes to the development of open source drivers for its devices.

[15]The progressive move of several firms to open source may have also been related to changes in technology that made open-source development more attractive. For example, the wide availability of hosting services, such as SourceForge and GitHub, lowered the cost of organizing and contributing to open-source projects.

[16]"Competing Against Amazon, Microsoft Adds Linux Support to Windows Azure Cloud," CMS Wire, June 7, 2012, available at http://www.cmswire.com/cms/information-management/competing-against-amazon-microsoft-adds-linux-support-to-windows-azure-cloud-015961.php.

11. Conclusion

In this paper, I study firm's openness and product-development decisions in imperfectly competitive environments with open and user innovation. The paper provides several testable results, which have direct managerial implications.

First, I show that product-market competition, the intensity of user innovation (number of contributions per user), and the size of the knowledge stock have a positive effect on openness decisions, and that firm size is negatively related to the openness degree and positively related to investments in product development.

These results can be tested with data on open-source projects, measuring openness as the proportion of compiled or source code released under an open-source license, measuring firm size as market share or the number of employees, and measuring the investment in product development with the lines of code of open-source projects. Part of this information is readily available in open-source development platforms, such as GitHub.

A good precedent for this type of research is Bonaccorsi, Giannangeli, and Rossi (2006). These authors present a survey of Italian software firms and show that most firms combine proprietary and OS products and receive revenues from traditional license fees as well as from open-source related services. They also examine the determinants of the degree of openness, and find that size (measured as the number of employees) is negatively correlated with openness. This paper provides a potential explanation for this finding.

Second, I show that openness decisions are strategic complements, which explains why firms usually react to higher openness by rivals by becoming more open, as in the cases of IBM versus JBoss, Facebook versus OpenSocial, Microsoft Windows versus Linux, Apple's iOS versus Android, and Toyota versus Tesla. To the best of my knowledge, this finding has not been tested empirically.

Finally, I show that compatibility and spillovers have a negative effect on openness, and that firms become more open as the number of competitors increases. As with the previous results, these findings can be tested empirically using data on open-source projects.

From a theoretical point of view, a limitation of the model is its static nature. A dynamic model could allow for the study of the timing of openness decisions. Existing dynamic models of open source consider a non-strategic community of developers (Casadesus-Masanell and Ghemawat, 2006; Athey and Ellison, 2010). An important exception is Tesoriere and Balletta (2017), who study a dynamic model with open-source and proprietary firms, but do not allow for user innovations. Developing a dynamic model with for-profit open-source firms and user innovation presents an interesting direction for future research.

Appendix A. Proofs

Proof of Proposition 1. In a symmetric equilibrium, it must hold that $s_1 = s_2 = 1/2$. It is straightforward to verify that these market shares satisfy equilibrium condition (7): substituting $s_i = 1/2$ on the right hand side, I obtain $F\left(\left(1 - 2\,\frac{1}{2}\right)\left(f(F^{-1}(1/2))^{-1} - \sigma\, x\right)\right) = F(0) = 1/2$. Thus, an equilibrium exists. Substituting $s_i = 1/2$ into (8), I obtain

$$\phi^* = 1 - \frac{1}{(2+\sigma)\, x\, f(0)}.$$

Given that there is only one solution ϕ^* that satisfies $s_i = 1/2$, there exists only one symmetric equilibrium. Finally, it is straightforward to show that ϕ^* increases with σ, x and $f(0)$. Results in the proposition follow. ■

Proof of Proposition 2. Let $H(s) = F\left((1-2\,s)\left(\frac{1}{f(F^{-1}(s))} - \sigma\, x\right)\right) - s$. By (7), there exists an equilibrium for all s such that $H(s) = 0$, and from proposition 1, $H(1/2) = 0$. The limits for $H(s)$ as s goes to 0 and 1 are $\lim_{s\to 0} H(s) = F(\infty) - 0 = 1 > 0$ and $\lim_{s\to 1} H(s) = F(-\infty) - 1 = -1 < 0$. Given that H is continuous and differentiable, by the intermediate value theorem, if $H'(1/2) > 0$ then there must exist $s \in (0, 1/2)$ and $s \in (1/2, 1)$ such that $H(s) = 0$. Differentiating $H(s)$, I obtain

$$H'(s) \;=\; f\left((1-2\,s)\left(\frac{1}{f(F^{-1}(s))} - \sigma\, x\right)\right)\left(-2\left(\frac{1}{f(y)} - \sigma\, x\right) - (1-2\,s)\,\frac{f'(F^{-1}(s))}{f(F^{-1}(s))^3}\right) - 1.$$

In equilibrium, $(1-2\,s)\left(\frac{1}{f(F^{-1}(s))}-\sigma\,x\right)=F^{-1}(s)$. Let $y(s)=F^{-1}(s)$, $y'(s)=\frac{1}{f(y)}$. Then,

$$\begin{aligned} H'(s) &= f(y)\left(-2\left(\frac{1}{f(y)}-\sigma\,x\right)-(1-2\,s)\,\frac{f'(y)}{f(y)^3}\right)-1, \\ &= -2\,(1-\sigma\,x\,f(y))-(1-2\,s)\frac{f'(y)}{f(y)^2}-1, \\ &= 2\,\sigma\,x\,f(y)-(1-2\,s)\frac{f'(y)}{f(y)^2}-3. \end{aligned}$$

If $\sigma\,x\,f(0)>3/2$, then $H'(1/2)=2\,\sigma\,x\,f(0)-3>0$ and there exist asymmetric equilibria. Necessity follows because $\sigma\,x\,f(0)<3/2$ implies $H'(s)<0$ in any equilibrium, which implies that the unique equilibrium is the symmetric one. To see this result, note that f is maximal at 0, and that $1-2\,s$ always has the same sign as $f'(y)$ ($s<1/2$ iff $y<0$). Therefore, in equilibrium, it holds that

$$H'(s)\;=\;2\,\sigma\,x\,f(y)-(1-2\,s)\frac{f'(y)}{f(y)^2}-3\;<\;2\sigma\,x\,f(0)-3.$$

Therefore, if $\sigma\,x\,f(0)<3/2$, then $H'(s)<0$ in any s such that $H(s)=0$. Finally, to see that in an asymmetric equilibrium the larger firm is less open than its rival, note that the ratio

$$\frac{1-\phi_i^*}{1-\phi_j^*}=\frac{\frac{s_i^*}{(1+\sigma s_i^*)x f(F^{-1}(s_i^*))}}{\frac{s_j^*}{(1+\sigma s_j^*)x f(F^{-1}(s_j^*))}}=\frac{\frac{1}{s_j^*}+\sigma}{\frac{1}{s_i^*}+\sigma}$$

is larger than 1 if $s_i^*>1/2>s_j^*$. Thus, $s_i^*>s_j^*$ implies $\phi_i^*<\phi_j^*$. The equilibrium price is equal to $(1-\phi_i^*)x$, so the larger firm sets a higher price. Given that the larger firm sets a higher price and has a larger market share, in equilibrium, it has larger profits. ■

Proof of Proposition 3. To determine whether actions are strategic complements, it suffices to check the sign of $\frac{\partial^2\pi_i}{\partial\phi_i\,\partial\phi_j}$ for optimal decisions given expectations. Using the definition of the marginal user y_i and (5), I obtain

$$\frac{\partial^2\pi_i}{\partial\phi_i\,\partial\phi_j}\;=\;x^2\,(1+\sigma s_j^e)\,f(y_i)\left(1-\frac{F(y_i)\,f'(y_i)}{f(y_i)^2}\right).$$

This expression is positive given that the increasing hazard ratio assumption implies $f'(y_i) < f(y_i)^2/F(y_i)$. Thus, openness degrees are strategic complements. Let $\phi_i^R(\phi_j) = \phi_i^R(\phi_j; s_i^e)$ be firm i's reaction function given expectations s_i^e. Applying the implicit function theorem on the first-order condition, I obtain

$$\frac{\partial \phi_i^R}{\partial \phi_j} = -\frac{\frac{\partial^2 \pi_i}{\partial \phi_i\, \partial \phi_j}}{\frac{\partial^2 \pi_i}{\partial \phi_i^2}} = -\frac{x^2\,(1+\sigma s_j^e)\, f(y_i)\left(1-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}\right)}{-x^2\,(1+\sigma s_i^e)\, f(y_i)\left(2-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}\right)} = \frac{1+\sigma s_j^e}{1+\sigma s_i^e}\,\frac{1-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}}{2-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}}.$$

An equilibrium is locally stable (Cournot, 1838) if the slope of $\phi_i^R(\phi_j)$ is larger than the slope of $\phi_j^R(\phi_i)$ in the (ϕ_i, ϕ_j) space (ϕ_i is the abscissa and ϕ_j is the ordinate). Given that the slope of $\phi_i^R(\phi_j)$ in the (ϕ_i, ϕ_j) space is $1/\frac{\partial \phi_i^R}{\partial \phi_j}$, the condition for stability is

$$\frac{\partial \phi_i^R}{\partial \phi_j}\,\frac{\partial \phi_j^R}{\partial \phi_i} = \frac{1-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}}{2-\frac{F(y_i)\, f'(y_i)}{f(y_i)^2}}\,\frac{1-\frac{F(y_j)\, f'(y_j)}{f(y_j)^2}}{2-\frac{F(y_j)\, f'(y_j)}{f(y_j)^2}} < 1.$$

Symmetry of the pdf implies that $F(y_i) = 1 - F(y_j)$, $f(y_i) = f(y_j)$, $f'(y_i) = -f'(y_j)$. Let $R(y_i) = \frac{F(y_i)\, f'(y_i)}{f(y_i)^2}$, and note that the increasing hazard ratio assumption implies $R(y_i) < 1$. Without loss, assume $f'(y_i) > 0$, which implies $R(y_i) > 0$ (otherwise, replace y_i by y_j and work with $F(y_j)$ and $R(y_j)$). Substituting into the above condition and operating, I obtain

$$(1 - R(y_i))\left(1 + R(y_i)\frac{1-F(y_i)}{F(y_i)}\right) < (2 - R(y_i))\left(2 + R(y_i)\frac{1-F(y_i)}{F(y_i)}\right),$$

which holds if $F(y_i) \in [0, 1]$ and $R(y_i) \in [0, 1]$. Thus, all equilibria are locally stable. ■

Proof of Proposition 4. Operating, I obtain

$$W = x + \sigma\, x\,(\phi_1 {s_1}^2 + \phi_2 {s_2}^2) - \mu\,(s_1 \log(s_1) + s_2 \log(s_2)) + I.$$

Substituting ϕ_i for the equilibrium expression (8) and letting $s_2 = 1 - s_1$, I obtain

$$\begin{aligned} W(s_1) \;&=\; x + \sigma\, x \left(\left(1 - \frac{\mu}{(1+\sigma s_1)\, x\,(1-s_1)}\right){s_1}^2 + \left(1 - \frac{\mu}{(1+\sigma(1-s_1))\, x\, s_1}\right)(1-s_1)^2\right) \\ &\quad -\mu\left(s_1 \log(s_1) + (1-s_1)\log(1-s_1)\right) + I. \end{aligned}$$

From Proposition 2, multiple equilibria exist only for $\sigma > \frac{3}{2\,x\,f(0)} = \frac{6\,\mu}{x}$. For σ close to $\frac{6\,\mu}{x}$, asymmetric equilibria are close to the symmetric equilibrium. Thus, it suffices to show that $W(s_1)$ increases as we move away from $s_1 = 1/2$. Differentiating and evaluating at $s_1 = 1/2$, it is straightforward to show that $W'(1/2) = 0$ and that

$$W''(1/2) \quad = \quad 4\left(\mu\left(\frac{32}{(\sigma+2)^3} + \frac{8}{\sigma+2} - 9\right) + \sigma\,x\right) > 0,$$

which is positive for $\frac{6\,\mu}{x} < \sigma < 1$. The result follows from continuity of W and (7). ∎

Proof of Proposition 5. Following the same steps as the proof of Proposition 1, it is straightforward to show that a symmetric equilibrium exists and that in such an equilibrium openness is

$$\phi^* = 1 - \frac{1}{(2+\sigma(1-\beta))\,x\,f(0)},$$

which is decreasing in β. In the symmetric equilibrium, profit is $\pi^* = (2\,(2+\sigma(1-\beta))\,f(0))^{-1}$, which is increasing in β. Finally, in the symmetric equilibrium, the quality of the open goods is

$$q^o = (1+\beta)\left(1+\frac{\sigma}{2}\right)x\left(1 - \frac{1}{(2+\sigma(1-\beta))\,x\,f(0)}\right),$$

which is increasing in β if $\beta < \frac{2+\sigma}{\sigma} - \sqrt{\frac{2(1+\sigma)}{\sigma^2\,x\,f(0)}}$. ∎

Proof of Proposition 6. By the implicit function theorem, a change in x_i has the following effect on the second-stage equilibrium market share:

$$\frac{\partial \hat{s}_i}{\partial x_i} \quad = \quad \frac{f(\hat{y}_i)\,(1+\sigma s_i^e)}{3-(1-2\hat{s}_i)\,\frac{f'(\hat{y}_i)}{f(\hat{y}_i)^2}}.$$

In the first stage, firm i chooses x_i to maximize

$$\pi_i = (1-\hat{\phi}_i)\,x_i\,s_i^d - \frac{c}{2}\,x_i = \frac{\hat{s}_i^2}{(1+\sigma\,s_i^e)\,f(F^{-1}(\hat{s}_i))} - \frac{c}{2}\,x_i.$$

The first-order condition is

$$\frac{\partial \pi_i}{\partial \hat{s}_i}\,\frac{\partial \hat{s}_i}{\partial x_i} - c\,x_i \;=\; \frac{\hat{s}_i}{f(\hat{y}_i)\,(1+\sigma s_i^e)}\left(2 - \hat{s}_i\,\frac{f'(\hat{y}_i)}{f(\hat{y}_i)^2}\right)\frac{f(\hat{y}_i)\,(1+\sigma s_i^e)}{3-(1-2s_i^*)\,\frac{f'(y_i^*)}{f(y_i^*)^2}} - c\,x_i \;=\; 0.$$

Imposing fulfilled expectations, I obtain the expression for x_i^* given in the text. To see that a symmetric equilibrium exists, note that $s_i^* = \frac{1}{2}$, $x_i^* = \frac{1}{3c}$, $i = 1, 2$ solves the equilibrium equations. To see that in an asymmetric equilibrium, the firm with the largest market share invests more than its rival, consider the ratio

$$\frac{x_i^*}{x_j^*} = \frac{s_i^*\left(2 - s_i^*\,\frac{f'(y_i^*)}{f(y_i^*)^2}\right)}{(1-s_i^*)\left(2+(1-s_i^*)\,\frac{f'(y_i^*)}{f(y_i^*)^2}\right)},$$

where $f(y_i^*) = f(y_j^*)$ and $f'(y_i^*) = -f'(y_j^*)$ follow from the symmetry of the pdf, and $s_j^* = 1 - s_i^*$ by definition. If $s_i^* > 1/2$, then $f'(y_i^*) < 0$, and the ratio is larger than 1. Finally, to see that $s_i^* > 1/2$ implies higher price and profits for firm i, note that price is $(1-\phi_i^*)\,x_i^* = \frac{s_i^*}{(1+\sigma\,s_i^*)\,f(F^{-1}(s_i^*))}$, which is increasing in s_i^*. The results follow. ■

Proof of Proposition 7. Follows the same steps as the proof of Proposition 1. ■

Proof of Proposition 8. The first-order condition is

$$-x\,s_i^d + (1-\phi_i)\,x\,f(y)\,\left((1+\sigma s_i^e)\,x + \alpha\,(\sigma\,x)^2\,\phi_i\right) = 0,$$

where $y = (1+\sigma s_i^e)\,\phi_i\,x + \frac{\alpha}{2}\,(\sigma\,\phi_i\,x)^2 - (1+\sigma(1-s_i^e))\,\phi_j\,x - \frac{\alpha}{2}\,\left(\sigma\,\phi_j\,x\right)^2$. The proof of existence follows the same steps as the proof of Proposition 1. In the symmetric equilibrium,

$$-x\,\frac{1}{2} + (1-\phi^*)\,x\,f(0)\,\left(\left(1+\sigma\frac{1}{2}\right)\,x + \alpha\,(\sigma\,x)^2\,\phi^*\right) = 0.$$

Solving for ϕ^*, I obtain

$$\phi^* = \frac{f(0)\,x\,(\sigma\,(2\,\sigma\,\alpha\,x-1)-2) + \sqrt{f(0)\,x^2\left(f(0)\,\left(\sigma+2\,\sigma^2\,\alpha\,x+2\right)^2 - 8\,\sigma^2\,\alpha\right)}}{4\,f(0)\,\sigma^2\,\alpha\,x^2},$$

which is positive whenever $f(0)\,(2+\sigma)\,x > 1$. The derivative with respect to α is

$$\frac{\partial \phi^*}{\partial \alpha} = \frac{4\sigma^2\alpha - f(0)(\sigma+2)\left(\sigma+2\sigma^2\alpha x+2\right) + \sqrt{f(0)}(\sigma+2)\sqrt{f(0)\,(\sigma+2\sigma^2\alpha x+2)^2 - 8\sigma^2\alpha}}{4\sqrt{f(0)}\sigma^2\alpha^2 x\sqrt{f(0)\,(\sigma+2\sigma^2\alpha x+2)^2 - 8\sigma^2\alpha}},$$

which is positive if $f(0)\,(2+\sigma)\,x > 1$. Thus, $\frac{\partial\phi^*}{\partial\alpha} > 0$ at any internal solution. User investments are $b_k^* = \sigma\,\phi^*\,x$, which are increasing in ϕ^*. This completes the proof. ■

Appendix B. IntelliJ data

The file size of the community and ultimate editions was obtained from http://jetbrains.com/idea/download/previous.html. File size corresponds to the Linux version of the latest stable release. The date for each version corresponds to the earliest release, obtained from http://blog.jetbrains.com/idea/category/releases/. Table 1 shows data for Figure 4.

Version	Date	Community	Ultimate	Openness
11.0	1-Feb-12	225.9	435.1	52%
11.1	28-Mar-12	240.9	453.1	53%
12.1	3-Apr-13	273.1	514.1	53%
13.0	3-Dec-13	343.5	595.1	58%
13.1	18-Mar-14	353.5	621.5	57%
14.0	5-Nov-14	376.1	653.0	58%
14.1	24-Mar-15	381.3	683.1	56%
15.0	1-Nov-15	452.6	794.7	57%
16.1	17-Mar-16	682.0	1021.9	67%
16.2	12-Jul-16	700.8	1057.7	66%
16.3	22-Nov-16	827.0	1232.8	67%
17.1	22-Mar-17	851.2	1264.8	67%

Table 1. IntelliJ community and ultimate editions' file size (MB)

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