COMPETITIVE STRATEGY FOR OPEN AND USER INNOVATION

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Abstract. I study openness and product-development decisions in competitive environments with open and user innovation. Firms may choose to open part of their knowledge or private information, so that it becomes freely accessible to users. Openness decisions are governed by a trade off between collaboration and appropriability: by becoming more open, a firm encourages user innovation, but at the same time, it hampers its ability to capture value. The benefits of openness and user innovation depend on users' expectations over the adoption rates of competing technologies. I find that openness decisions may interact with users' expectations to generate asymmetric market outcomes, in which case large firms are less open than small firms. I also find that firms react to increased openness from rivals by becoming more open. Firms' strategic actions have an *expectations effect*, which affects the incentives to invest in product development and leads to second-mover advantages. Finally, I find that compatibility and spillovers have a negative effect on openness, and that firms become more open as the number of competitors increases.

Keywords: Competitive Strategy, Open Innovation, User Innovation, Openness Choice, Asymmetric Equilibria, Expectations Effect, Second-Mover Advantages, Appropriability, Open-Source Software, Network Effects.

1. Introduction

An important question for competitive and technology strategy is whether firms should follow open or closed approaches towards product development and intellectual-property management. One of the most important benefits of openness is that it enables and facilitates user innovation (von Hippel, 1988, 2005; Morrison, Roberts, and von Hippel, 2000).[1] For example, the success of open standards, such as W3C, USB, and TCP/IP, depends on the contributions and feedback of standard's adopters (Rysman and Simcoe, 2008), and open-source software projects, such as Apache and Linux, rely on the contributions of users as diverse as independent developers, non-profit organizations, and

Date: December 2, 2016.

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[1]Chesbrough and Appleyard (2007) define openness as the "pooling of knowledge for innovative purposes where the contributors have access to the inputs of others and cannot exert exclusive rights over the resultant innovation." This definition encompasses ideas such as open innovation (Chesbrough, 2003), collective invention (Allen, 1983), and free and open-source software (Raymond, 1999; Stallman, 2002).

commercial firms.[2] On the other hand, openness generally lowers a firm's ability to capture value (Arrow, 1962). For example, releasing part of a firm's source code under an open-source license may cannibalize sales of the firm's proprietary offerings, and licensing a firm's technology to a client or rival may lead to undesired knowledge disclosures (Anton and Yao, 1994; Hellmann and Perotti, 2011). Therefore, openness leads to a *trade off between collaboration and appropriability*, which affects firms' decisions to open technologies (West, 2003; Henkel, 2006; Henkel, Schöberl, and Alexy, 2014).

In a competitive environment, firms' openness decisions are also affected by strategic considerations. In the mid 2000s, IBM's proprietary product WebSphere Application Server was facing strong competition from JBoss's open-source product JBoss Application Server in the middleware market.[3] IBM reacted by buying JBoss's main rival – software developer Gluecode– and releasing all of its source code under an open-source license. JBoss's CEO, Marc Fleury, complained that IBM's intention was to "kill JBoss," and that Gluecode would hurt sales of IBM's WebSphere as much as JBoss's.[4] Thus, IBM responded to competition from a firm that was more open by becoming more open, even though it meant creating competition for its own proprietary product. Likewise, Facebook decided to open part of its platform's source code as a response to OpenSocial –an open-source platform used by Google, MySpace and LinkedIn–[5] and Microsoft became more open after the success of open products such as Linux and Chrome.

Open and user innovation have been extensively studied empirically, but formal theoretical analyses are relatively scarce.[6] Consequently, there is a call to put open and user innovation on firmer theoretical grounds (West, Salter, Vanhaverbeke, and Chesbrough, 2014). I contribute to this literature by studying a formal game-theoretical model of product-market competition with open and user innovation.

From a methodological point of view, the paper links open and user innovation to the literature on network effects: because users provide innovations that benefit other consumers, the willingness to adopt the product of one firm will depend on the total number of users.[7] Moreover, given that users can innovate more when a firm opens

[2]Bonaccorsi and Rossi (2004), Shah (2006), Lakhani and Wolf (2005), and Roberts, Hann, and Slaughter (2006) show that user need is one of the main motivations for contributing to open-source projects.
[3]JBoss was later acquired by Red Hat.
[4]"Open Source Smack-Down," Forbes, June 15 2005, http://www.forbes.com/2005/06/15/jboss-ibm-linux_cz_dl_0615jboss.html, accessed December 2, 2016.
[5]"Facebook open-sources a 'significant part' of its platform," Cnet, June 2 2008, http://www.cnet.com/news/facebook-open-sources-a-significant-part-of-its-platform/, accessed December 2, 2016.
[6]See Section 2 for a detailed review of empirical and theoretical works on open and user innovation.
[7]Oh and Jeon, 2007, present evidence of membership herding in open-source projects, which is consistent with the existence of network effects.

more of its knowledge, *the choice of openness determines the intensity of network effects*. This mechanism is absent in more traditional studies of network effects.

The paper has two main results. First, I show that firms with a larger market share are less open and have larger profits than smaller firms. If users expect that a firm will have a larger user base, then they expect that firm to benefit more from user innovation. Thus, the firm can lower its openness degree and increase its price without loosing too many customers. In equilibrium, large and small firms make different uses of the *intensive and extensive margins*: large firms capture a larger number of users, but have a smaller number of contributions per user. This result is consistent with the findings of Bonaccorsi, Giannangeli, and Rossi (2006), who show that firm size is negatively correlated with openness. The result also explains the general observation that large firms, such as IBM in the middleware market, tend to be less open than small firms, such as JBoss. Interestingly, Bonaccorsi, Giannangeli, and Rossi find their own result surprising, and provide no theoretical explanation for it. To the best of my knowledge, this is the first paper providing a formal link between firm size and openness.

Second, I show that firms' actions affect user expectations, which in turn affect the desirability of making strategic commitments. This link has so far been understudied by the literature of open and user innovation. In particular, I find that openness decisions lead to second mover advantages and affect the incentives to invest in product quality.

The first result follows from strategic complementarity (Bulow, Geanakoplos, and Klemperer, 1985) and the expectation-formation mechanism. Because openness decisions are strategic complements, the first mover has incentives to choose a smaller openness degree to induce its rival to be less open as well. For fixed user expectations, the second mover ends up being more open and having a larger equilibrium market share than the first mover (this effect was first studied by Gal-Or, 1985). Users anticipate this effect and adjust expectations accordingly, which has a negative effect on users' expectations. Therefore, the game exhibits second-mover advantages for two reasons: because actions are strategic complements , and because moving first has a *negative expectations effect*.

The second result is due to the fact that quality differences affect the set of market equilibria that are consistent with fulfilled expectations. Interestingly, the effect of a higher investment on user expectations may be *positive or negative*. For example, when investment is costly relative to the intensity of product-market competition, a larger investment has a negative effect on expectations. As a result, firms invest less in product development than what they would invest if investments did not affect users' expectations. I also find that, in this case, equilibrium investment decreases as the number of contributions per user increases, which is an unexpected result, given that user investments are complementary to firm investments.

The paper contributes to the literature on open and user innovation, by providing a theoretical model to study openness decisions under competition, and by studying the effect of users' expectations on the desirability of making strategic commitments. In particular, the paper contributes to two of the most important avenues for future research in open innovation identified by West et al. (2014), namely, (i) to understand the impact of appropriability on openness decisions and innovation output, and (ii) to link open innovation to management and economics. In the conclusion, I explain additional results, describe ways to test them empirically, discuss the paper's managerial implications, and present directions for further research.

2. Literature review

Most works studying open innovation are empirical papers or case studies (see, for example, Chesbrough, 2003; Christensen, Olesen, and Kjær, 2005; Chesbrough and Crowther, 2006; Laursen and Salter, 2006). Two notable exceptions are Almirall and Casadesus-Masanell (2010) and Gambardella and Panico (2014). Almirall and Casadesus-Masanell (2010) present an NK model of fitness landscapes to study the trade off between adoption and appropriability. In contrast with this paper, they focus on value creation, instead of value capture, and do not consider strategic interaction between firms. Gambardella and Panico (2014) present a game-theoretic model to study the desirability of transferring innovation decision rights to external agents, so they focus on the internal organization of the firm, rather than on the effects of external competition.

The paper is also related to the literature on open-source software. Bessen (2006) and Niedermayer (2013) show that open source allows firms to overcome organizational and contractual problems. I focus on the effects of openness on external competition rather than on the internal organization of the firm. Johnson (2002), Harhoff, Henkel, and von Hippel (2003), Bessen (2006), and Reisinger, Ressner, Schmidtke, and Thomes (2014) study open source as a mechanism to develop a public good with private investments. In the basic model of Section 3, competitors' use non-compatible independent knowledge pools, so knowledge is not a public good. In Section 7, I study the effect of partial compatibility and spillovers, in which case disclosed knowledge becomes a public good. I show that spillovers reduce equilibrium openness levels because they lead to free riding, and also because they allow firms to commit to compete less aggressively to attract users.

Johnson (2002), Casadesus-Masanell and Ghemawat (2006), Polanski (2007) and Athey and Ellison (2010) analyze the interaction between open source and user innovation. Johnson (2002) and Polanski (2007) focus on developers' incentives and do not consider for-profit firms. Casadesus-Masanell and Ghemawat (2006) and Athey and Ellison (2010) study models of competition between a for-profit proprietary firm and a non-strategic community of developers. In contrast, I study the decisions of for-profit firms, and how they interact with users' incentives and expectations. Llanes and de Elejalde (2013) study the decision of for-profit firms to become open source or proprietary, but they do not study partial knowledge disclosures and do not consider user innovation.

Finally, Casadesus-Masanell and Llanes (2011) study the decision of a for-profit firm to release part of its software under an open-source license, as it competes with a pre-existing non-strategic open-source project. As in this paper, the firm's disclosed knowledge interacts with users' innovations. In contrast with that paper, I study competition between two or more for-profit firms, and focus on a different set of issues. Specifically, my paper has the following contributions. First, I show that strategic interaction has important effects on firms' openness decisions. For example, I show that the optimal reaction to an increase in the openness level of a rival is to become more open. Second, I examine the role of users' expectations in generating asymmetric market outcomes, and study the consequences on firms' openness decisions and profits. Third, I study the desirability of making strategic commitments, and show that the endogeneity of user expectation formation leads to second-mover advantages. Fourth, I study firms' product development incentives, and show that investment may have an expectations effect, which may be positive or negative. Finally, I study the effects of compatibility, spillovers, and market entry on openness decisions.

3. The model

Two firms, $i = 1, 2$, develop and sell products to a continuum of users with unit mass. User demands depend on prices and on the knowledge, ideas, and quality investments embedded in each product. Firms can protect their knowledge with intellectual property rights or secrecy, but they can also choose to disclose (open) part of their knowledge so that it becomes freely accessible to users. Users can use the firm's disclosed information to modify and improve the firm's products through *user innovation*, which benefits the firm as these innovations become available to other users. However, users can use the disclosed information to build their own version of the firm's product, which sets a limit to the price the firm can charge for the product that uses its open and proprietary knowledge. Thus, when choosing its openness degree, a firm faces a *trade off between collaboration and appropriability*: greater openness benefits the firm because other agents contribute to increasing the firm's knowledge, but it also reduces the firm's ability to capture value.

The model describes the fundamental trade off at play in a variety of cases. For example, by releasing part of its source code under an open-source license, a software firm allows for greater user innovation, but at the same time creates competition for the product that includes all of its source code. Likewise, a firm contributing its patents to an open standard benefits when standards' adopters provide feedback and develop complementary products, but may have to commit to license its patents on FRAND terms (Fair, Reasonable, and Non-Discriminatory), which sets a cap on the royalty fees it can charge. Also, a firm selling an input to downstream product manufacturers benefits if manufacturers complement the firm's technologies with their own innovations, but in order to benefit from these innovations the firm may have to disclose private information that manufacturers can use to develop their own substitute inputs.

The quality of the *proprietary good* of firm i (the product that includes the firm's open and proprietary knowledge) is $q_i^p = x_i + z_i$, where x_i represents firm i's stock of knowledge and z_i represents the innovations contributed by firm i's users. The quality of the product based on firm i's open knowledge is $q_i^o = \phi_i \, x_i + z_i$, where ϕ_i is the fraction of knowledge disclosed by the firm (openness degree). I will refer to this product as firm i's *open product.*

In the case of a software program, for example, x_i could represent the number of lines of the source code of the firm, ϕ_i the fraction of source code released under the open-source license, and z_i the number of lines of code contributed by user/developers.

User innovation is $z_i = \sigma \, \phi_i \, x_i \, s_i^e$, where $\sigma \in [0, 1]$ represents the intensity of user innovation (contributions per user), and s_i^e is the expected measure of users of the products of the firm. This simple functional form captures two effects: (i) users can innovate more if they can access more knowledge (intensive margin), and (ii) more users imply more user innovation (extensive margin). I am implicitly assuming that individual user investments are exogenously given (each user innovates a fraction σ on top of the knowledge she can access, and total user innovation is the sum of the innovations of each user). In Section 10 I extend the model to allow for endogenous user investments.

Following Katz and Shapiro (1985), I assume that players take the expected size of the network (s_i^e) as given when making their decisions, and that such expectations are fulfilled in equilibrium ($s_i^* = s_i^e$).

Each user chooses a product to maximize her indirect utility. User k's indirect utility for consuming firm i's proprietary product is

$$v^p_{ik} \;=\; q^p_i - p_i + \varepsilon_{ik}, \tag{1}$$

and her indirect utility for consuming firm i's open product is

$$v^o_{ik} \;=\; q^o_i + \varepsilon_{ik}, \tag{2}$$

where p_i is the price of the firm i's proprietary product and ε_{ik} is an idiosyncratic taste parameter. Let s^p_i and s^o_i be the share of users choosing the proprietary and open products of firm i, and let $s_i = s^p_i + s^o_i$ be the total share of users of firm i. Users choose the proprietary good when they are indifferent between the open and proprietary products of a firm.

The taste differential $\omega_k = \varepsilon_{1k} - \varepsilon_{2k}$ has a cumulative density function F. The probability density function, $f = F'$, is symmetric, strictly positive in the support $(-\infty, \infty)$, and centered around zero, with $f'(\omega) > 0$ for $\omega < 0$, $f'(0) = 0$, and $f'(\omega) < 0$ for $\omega > 0$. The hazard function, $h = f/(1-F)$, is increasing. Let F^{-1} be the inverse function of the cumulative density function. Symmetry implies $f(\omega) = f(-\omega)$ and $F(\omega) = 1 - F(-\omega)$.

The products of different firms are differentiated vertically and horizontally, but the open and proprietary goods of the same firm are differentiated only along the vertical dimension: given that they are based on the same basic knowledge, they share horizontal characteristics, and the only difference between the two goods is that the proprietary product incorporates more knowledge than the open product.

The model is a two-stage non-cooperative game. In the first stage, firms choose ϕ_i and p_i simultaneously to maximize profits $\pi_i = p_i\, s^p_i$, taking expectations s^e_i as given. In the second stage, users observe firms' prices and qualities, and choose their preferred variety. In equilibrium, expectations are fulfilled ($s^*_i = s^e_i$). The equilibrium concept is subgame-perfect Nash equilibrium with fulfilled expectations.

4. Equilibrium

Substituting q^p_i and q^o_i into (1) and (2), I obtain

$$\begin{aligned} v^p_{ik} &= x_i + \sigma\,\phi_i\,x_i\,s^e_i - p_i + \varepsilon_{ik}, \\ v^o_{ik} &= \phi_i\,x_i + \sigma\,\phi_i\,x_i\,s^e_i + \varepsilon_{ik}. \end{aligned} \tag{3}$$

Given that the open and proprietary products of a firm are differentiated only along the vertical dimension, and that users are homogeneous in their willingness to pay for quality, $s_i^p = 0$ if $\phi_i\, x_i > x_i - p_i$, and $s_i^o = 0$ if $\phi_i\, x_i \leq x_i - p_i$. In equilibrium it must hold that $p_i \leq (1 - \phi_i)\, x_i$. Otherwise firm i would make no sales of its proprietary good. Also, if $p_i < (1 - \phi_i)\, x_i$ the firm can raise ϕ_i without changing p_i, which increases user innovation and the demand for its proprietary good, while keeping $s_i^o = 0$. In equilibrium, $p_i = (1 - \phi_i)\, x_i$, so I focus on the choice of openness degree. Substituting p_i in (3) and operating, I obtain

$$v_{ik}^p = (1 + \sigma s_i^e)\, \phi_i\, x_i + \varepsilon_{ik}.$$

As explained above, when choosing their openness level, firms face a *trade-off between collaboration and appropriability*: by increasing ϕ_i, a firm encourages user innovation, but it also raises competition between its open and proprietary offerings, thus lowering the price it can charge for its proprietary product.

Users observe qualities and prices and choose their preferred product. The demand of firm i's proprietary product is

$$\begin{aligned} s^d(\phi_i, \phi_j, x_i, x_j, s_i^e) &= \Pr\Big((1 + \sigma s_i^e)\, \phi_i\, x_i + \varepsilon_{ik} \geq (1 + \sigma(1 - s_j^e))\, \phi_j\, x_j + \varepsilon_{jk}\Big), \\ &= F\Big((1 + \sigma s_i^e)\, \phi_i\, x_i - (1 + \sigma(1 - s_j^e))\, \phi_j\, x_j\Big). \end{aligned} \tag{4}$$

In the first stage, firms choose $\phi_i \in [0, 1]$ to maximize

$$(1 - \phi_i)\, x_i\, s^d(\phi_i, \phi_j, x_i, x_j, s_i^e),$$

taking s_i^e as given. In equilibrium, expectations are fulfilled ($s_i^* = s_i^e$). Proposition 1 shows the equilibrium of the game. For the moment, I study a case in which private investments in product quality are symmetric, $x_1 = x_2 = x$, and postpone the discussion of asymmetric investments for Section 8. All proofs are in the Appendix.

Proposition 1 (Equilibrium)**.** *A fulfilled-expectations equilibrium exists. The number of equilibria is finite and odd. In equilibrium, the market share of firm i solves*

$$s_i^* = F\left((1 - 2s_i^*)\left(\frac{1}{f(F^{-1}(s_i^*))} - \sigma x\right)\right),$$

and equilibrium openness is

$$\phi_i^* = 1 - \frac{s_i^*}{(1 + \sigma s_i^*)\, x\, f(F^{-1}(s_i^*))}.$$

A symmetric equilibrium ($s_1^* = s_2^* = 1/2$*) exists and is unique.*

In the symmetric equilibrium, the openness degree of both firms is

$$\phi^* = 1 - \frac{1}{(2+\sigma)\, x\, f(0)}.$$

Thus, in equilibrium firms become proportionally more open if user innovation is more important, competition is stronger (the slope of the demand function in equilibrium, $f(0)$, is larger), or the initial stock of knowledge is larger. To understand why openness increases with σ and x, note that if user innovation or the knowledge stock is larger, users can introduce more innovations for a given openness level. Thus, an increase in σ or x increases the sensitivity of user innovation (and profits) to changes in the openness degree, which causes firms to become more open. To see why equilibrium openness increases with the intensity of product-market competition, note that when product-market competition increases, the demand-stealing effect of encouraging more user innovation increases, and thus firms' compete more fiercely for users. As a result, firms end up being more open than what they would be if they were more isolated from competition.

In addition to the symmetric equilibrium, there may exist asymmetric equilibria, as the following proposition shows.

Proposition 2 (Asymmetric equilibria)**.** *If $\sigma\, x\, f(0) > 3/2$, there exist at least two asymmetric equilibria in which one firm has a larger market share than its rival. In any asymmetric equilibrium, the larger firm is less open and has higher profits than its rival. If $\sigma\, x\, f(0) \leq 1$, the unique equilibrium is symmetric.*

Proposition 2 shows that if user innovation, the knowledge stock, and product market-competition are large enough, there exist asymmetric equilibria in which one of the firms is larger than its rival. Ex-post asymmetries arise even if firms are ex-ante symmetric in terms of product characteristics, and are caused by the interdependence of user-adoption decisions: if a firm has more favorable expectations over its user-adoption rates, users will expect that firm to benefit more from user innovation, and thus they will be more inclined to chose the firm's product over its rival's (holding everything else constant), which in turn justifies the more favorable expectations that users had in the first place.

In the asymmetric equilibrium, firms with a larger market share are less open and have larger profits than smaller firms. If users expect that a firm will have a larger user base, then they expect that firm to benefit more from user innovation for a given openness degree. Thus, the firm can lower its openness degree and increase price without loosing too many customers. Large and small firms make different uses of the *intensive and extensive margins*: the larger firm will have a smaller number of contributions per user, but it will have a larger number of users. This results helps explain why larger firms, such as IBM in the middleware market, tend to be more closed than smaller firms, such as JBoss. This result is consistent with Bonaccorsi et al. (2006), which shows that firm size is negatively correlated with firm openness.

5. Logistic distribution

To illustrate the results of the previous section, consider an example based on the logistic distribution. The cumulative distribution function is

$$F(\omega) = \frac{1}{1 + \exp(-\omega/\mu)},$$

the density function is $f = F(1 - F)/\mu$, and the variance is $\pi^2 \mu^2/3$. The larger μ, the larger the dispersion in user tastes and the lower the intensity of competition.

Equilibrium condition (1) can be arranged in the following way:

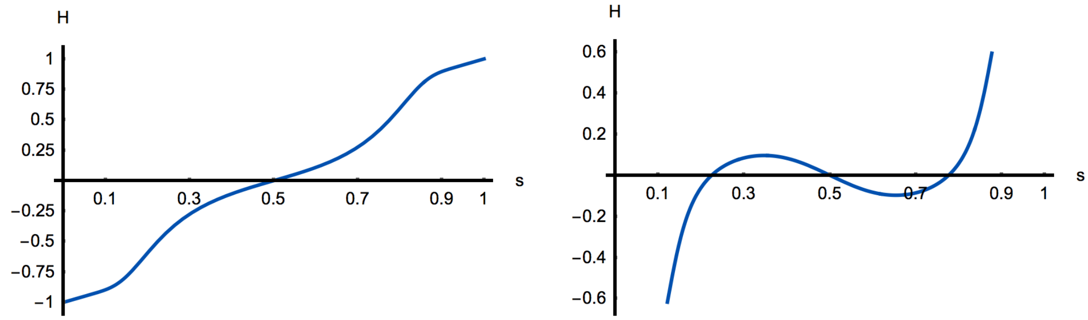
$$H(s) = s - \left[1 + \exp\left((1 - 2s)\left(\frac{1}{s(1-s)} - \frac{\sigma x}{\mu}\right)\right)\right]^{-1} = 0.$$

The equilibrium depends on $\sigma x/\mu$, which compares the importance of user innovation and the knowledge stock to the intensity of competition. When $\sigma x/\mu < 6$ the unique equilibrium is symmetric. If $\sigma x/\mu \geq 6$ there are three equilibria: the symmetric equilibrium and two asymmetric equilibria in which one of the firms has a larger market share. The openness degree in the symmetric equilibrium is

$$\phi^* = 1 - \frac{4\mu}{(2 + \sigma)\, x}.$$

Figure 1a shows the graph of $H(s)$ for $\sigma = 0.2$, $x = 20$ and $\mu = 1$. In this example, $\sigma x/\mu = 4$, so there is only one equilibrium. Openness degree and profits are $\phi^* = 0.91$ and $\pi^* = 0.91$.

Figure 1b shows $H(s)$ for $\sigma = 0.2$, $x = 40$ and $\mu = 1$. In this example, $\sigma x/\mu = 8$, so there are three equilibria: $\{s_1^* = 0.22, s_2^* = 0.78\}$, $\{s_1^* = 0.5, s_2^* = 0.5\}$, and $\{s_1^* = 0.78, s_2^* = 0.22\}$. As for the openness degree and profits, $\phi_i^* = 0.97$ and $\pi_i^* = 0.28$ for $s_i^* = 0.22$, $\phi_i^* = 0.95$ and $\pi_i^* = 0.91$ for $s_i^* = 0.5$, and $\phi_i^* = 0.9$ and $\pi_i^* = 2.99$ for $s_i^* = 0.78$. Therefore, in the asymmetric equilibrium, the small firm is more open and has lower profits than in the symmetric equilibrium, and the large firm is less open and has higher profits than in the symmetric equilibrium.



(A) Unique equilibrium (B) Multiple equilibria

FIGURE 1. Logistic distribution

6. Sequential play

In this section, I study the incentives to open technologies when firms choose their openness degrees sequentially. More importantly, I analyze whether firms have incentives to commit to choose their openness degree before their rivals.

Openness decisions are generally difficult to revert, which means that they can be used as strategic commitments. Clearly, once information is disclosed, it is not possible to make it private again as was emphasized by Arrow (1962). Moreover, Baldwin and Clark (2006) and MacCormack, Rusnak, and Baldwin (2006) show that openness decisions have important effects on architectural design, and Bonaccorsi et al. (2006) show that they affect organizational routines, all of which are costly to revert.

The game becomes a three-stage non-cooperative game. In the first stage, firm 1 chooses its price and openness degree. In the second stage, firm 2 observes firm 1's choices and then chooses its price and openness degree. In the third stage, users observe prices and product characteristics and choose which product they will consume. As in the previous game, I follow Katz and Shapiro (1985) and assume that players take expectations over network sizes as given when choosing their actions, but that such expectations are fulfilled in equilibrium. Proposition 3 characterizes the equilibrium of the game.

Proposition 3 (Sequential play). *A fulfilled-expectations equilibrium exists. The number of equilibria is finite and odd. The equilibrium market share of the first mover is defined implicitly by*

$$s_1^* \quad = \quad F\left((1-2s_1^*)\left(\frac{1}{f(F^{-1}(s_1^*))} - \sigma x\right) - \frac{s_1^*}{f(F^{-1}(s_1^*))}\, g(F^{-1}(s_1^*))\right),$$

where $g(\omega) = h'(\omega)/h(\omega)^2 > 0$. The market share of the second mover is $s_2^* = 1 - s_1^*$. The openness degrees of the first and second movers are

$$\begin{aligned}\phi_1^* &= 1 - \frac{s_1^*}{(1+\sigma s_1^*)\, x\, f(F^{-1}(s_1^*))}\,(1 + g(F^{-1}(s_1^*)),\\ \phi_2^* &= 1 - \frac{s_2^*}{(1+\sigma s_2^*)\, x\, f(F^{-1}(s_1^*))}.\end{aligned}$$

Actions (openness degrees) are strategic complements. An equilibrium with $s_1^* < \frac{1}{2} < s_2^*$ exists. If $\sigma\, x\, f(0) \leq 1$ all equilibria have $s_1^* < \frac{1}{2} < s_2^*$. In any equilibrium with $s_1^* < \frac{1}{2} < s_2^*$, the first mover has lower profits than the second mover.

Proposition 3 shows that openness degrees, ϕ_i, are strategic complements (Bulow et al., 1985): an increase in the openness degree of one firm leads to an increase in the openness degree of its rival. This result provides formal support to the observation that firms usually respond to an increased openness by rivals by becoming more open, as the cases of IBM versus JBoss and Facebook versus OpenSocial mentioned in the introduction show.

Strategic complementarity has important implications for the analysis of sequential play. Gal-Or (1985) shows that when actions are strategic complements, the game has *second-mover advantages*, which means that the second mover has larger profits than the first mover. However, in the games that Gal-Or (1985) studies, users' expectations do not play any role because users do not care about the size of competing networks. In contrast, in the game studied here, sequential play has an additional effect on profits through its effect on expectations formation.

To see this, consider the equilibrium with $s_1^* < 1/2 < s_2^*$ closest to the symmetric distribution $s_1 = s_2 = 1/2$. Since $\partial\pi_1/\partial\phi_2 < 0$ (an increase in the action of one firm lowers the profits of its rival), the first mover chooses a lower action than what it would choose in a simultaneous-move game, in order to induce its rival to choose a lower action too. Given fixed expectations, the second mover ends up being more open and having a larger equilibrium market share than the first mover.[8] Therefore, firm 1 has a tendency to choose a smaller openness degree than firm 2. Users anticipate this effect and adjust expectations accordingly, which has a negative effect on users' expectations over the first mover's market share.

Thus, the game has second-mover advantages for two reasons: because actions are strategic complements, and because moving first leads to a *negative expectations effect.*

[8]This result is evident when comparing ϕ_1^* and ϕ_2^*: the second term of ϕ_1^* is multiplied by $1+g(F^{-1}(s_1^*) > 1$, but ϕ_2^* is not multiplied by a similar term).

The managerial implication of this result is that it is important to take users' expectations into account when determining the desirability of making strategic commitments, such as committing to choose the openness degree before rivals.

It is important to understand what second-mover advantages mean in this game with network effects. Due to the existence of multiple equilibria, there may exist equilibria in which the first mover has a larger market share than the second mover ($s_1^* > 1/2 > s_2^*$). Thus, it is possible for the first mover to have larger profits than the second mover, but the important fact is that the second mover benefits *more* from sequential play than the first mover, when comparing a particular equilibrium with the closest equilibrium of a simultaneous-move game.[9]

The expectations effect has two additional implications. First, when $s_1^* < 1/2 < s_2^*$, the expectations effect counteracts the result stated in Proposition 1 that large firms are less open than small firms. To see this, consider the ratio

$$\frac{1-\phi_1^*}{1-\phi_2^*} = \frac{\frac{1}{s_2^*}+\sigma}{\frac{1}{s_1^*}+\sigma}\left(1+g(F^{-1}(s_1^*)\right).$$

Given that $1+g(F^{-1}(s_1^*) > 1$, it is now possible that $\phi_1^* < \phi_2^*$ when $s_1^* < 1/2 < s_2^*$. On the other hand, it is straightforward to see that sequential play strengthens the result that $\phi_1^* > \phi_2^*$ when $s_1^* > 1/2 > s_2^*$.

Second, the first mover may have smaller profits in a sequential game than in a simultaneous-move game. This result is not possible in a game without network effects (as in Gal-Or, 1985), in which expectations effects are absent, and thus a first mover always benefits from playing sequentially. Without network effects, the first mover knows that the second mover will choose an action in its best-response function, so it could induce the equilibrium of the simultaneous-move game if it wanted to do so. Thus, if the first mover is choosing something different in equilibrium, it must be because it is better off than what it would be in the simultaneous-move game. In contrast, in a game with network effects, the expectations effect implies that the first mover has less-favorable expectations when playing sequentially than when playing at the same time as the second mover. If this negative expectations effect is large enough, the first mover may lose from playing first, in comparison with a simultaneous-move game.[10]

[9] When comparing equilibria of the sequential game with that of a simultaneous-move game, assume that the model fundamentals are such that the number of equilibria in both games is the same.

[10] The expectations effect can be made as large as wanted by increasing $g = h'/h^2$.

7. Partial compatibility and spillovers

In this section, I study the effects of knowledge complementarity and spillovers. For example, in the software industry, competing firms may use similar programming languages and architectures, which may allow them to utilize some of their rival's open-source code. Fershtman and Gandal (2011) find evidence of knowledge spillovers in open-source projects.

Assume that firms may use a fraction β of the knowledge disclosed by their rivals. The quality of the proprietary good of firm i is $q_i^p = x + \beta\,\phi_j\,x + z_i$ and the quality of the open good of firm i is $q_i^o = \phi_i\,x + \beta\,\phi_j\,x + z_i$, where x is firm i's knowledge, z_i is user innovation, and ϕ_i is the openness degree of firm i. User innovation is given by $z_i = \sigma\,\phi_i\,x\,s_i^e + \sigma\,\beta\,\phi_j\,x\,s_j^e$. Therefore, β measures the extent of spillovers between the products of the two firms. Proposition 4 shows the equilibrium of the game.

Proposition 4 (Partial compatibility and spillovers)**.** *A fulfilled-expectations equilibrium exists. The number of equilibria is finite and odd. The equilibrium market share of firm i is defined implicitly by*

$$s_i^* \quad = \quad F\left((1-2s_i^*)\left(\frac{1}{f(F^{-1}(s_i^*))} - \sigma(1-\beta)x\right)\right),$$

and equilibrium openness is

$$\phi_i^* \quad = \quad 1 - \frac{s_i^*}{(1+\sigma(1-\beta)s_i^*)\,x\,f(F^{-1}(s_i^*))}.$$

A symmetric equilibrium ($s_1^* = s_2^* = 1/2$) exists and is unique. In the symmetric equilibrium, firms' profits are increasing in β. If $\sigma\,(1-\beta)\,x\,f(0) > 3/2$, there exist at least two asymmetric equilibria in which one firm has a larger market share than its rival. In the asymmetric equilibrium, the larger firm is less open and has higher profits than its rival. If $\sigma\,(1-\beta)\,x\,f(0) \leq 1$, the unique equilibrium is symmetric.

Partial compatibility has two effects on openness decisions. First, a firm's relative advantage from opening technologies is reduced, given that rivals take advantage of the knowledge disclosed by the firm. This effect makes firms less willing to open their technologies. Second, spillovers reduce the demand-stealing effect of improving the quality of a firm's product. Thus, firms compete less fiercely to capture users, and are less inclined to open their technologies. In the symmetric equilibrium, compatibility leads to less openness and user innovation, higher prices and profits for firms, and lower surplus for users.

8. Firm investment in product quality

In this section, I allow for differences in firms knowledge stocks or investments in product quality. Demand is given by (4). Proposition 5 shows the equilibrium of the game for fixed levels of x_1 and x_2.

Proposition 5 (Vertical product differentiation). *A fulfilled-expectations equilibrium exists. The number of equilibria is finite and odd. The equilibrium market share of firm 1 is defined implicitly by*

$$s_1^* \;=\; F\left((1-2s_1^*)\left(\frac{1}{f(F^{-1}(s_1^*))}-\sigma x_2\right)+(1+\sigma s_1^*)\,(x_1-x_2)\right),$$

and the market share of firm 2 is $s_2^* = 1 - s_1^*$. The openness degrees firms 1 and 2 are

$$\phi_1^* \;=\; 1-\frac{s_1^*}{(1+\sigma s_1^*)\,x_1\,f(F^{-1}(s_1^*))},$$

$$\phi_2^* \;=\; 1-\frac{s_2^*}{(1+\sigma s_2^*)\,x_2\,f(F^{-1}(s_2^*))}.$$

If $x_1 > x_2$ ($x_1 < x_2$) an equilibrium with $s_2^* < \frac{1}{2} < s_1^*$ ($s_2^* > \frac{1}{2} > s_1^*$) exists, and, in such equilibrium, firm 1 has higher (lower) profits than firm 2.

Proposition 5 shows that when one firm has a larger knowledge stock, the equilibrium closest to the symmetric distribution $s_1 = s_2 = 1/2$ becomes more favorable to that firm, in terms of market share and profits. In what follows, I focus on this equilibrium, and explore the effects of endogenous product development. For simplicity, I assume that firms follow a constant pricing rule, so that $p_i = (1-\phi_i)\,x_i$ (this assumption is without loss of generality for studying the equilibrium of the game, but simplifies the computation of strategic effects).

In particular, I consider the following four-stage non-cooperative game. In the first stage, firms choose quality investments x_i. In the second stage, users form expectations s_i^e. In the third stage, firms choose openness degrees. In the fourth stage, users observe prices and product characteristics and choose which product they want to consume. Product-development cost is $c\,x_i^2/2$, where c is a positive parameter. Assume that c is large enough so that second-order conditions are satisfied.

Before studying the subgame-perfect Nash equilibrium of the game, I study the effects of a change in x_1 on firm 1's profits in the continuation game. I divide the total effect in three: direct effect, strategic effect, and expectations effect. The *direct effect* is the effect of a change in x_1 on profits assuming that equilibrium actions ϕ_1^*, ϕ_2^* and users' expectations

s_i^e remain constant. The *strategic effect* is the effect of a change in x_1 on profits caused by a change in equilibrium actions ϕ_1^*, ϕ_2^*, assuming that users' expectations s_i^e remain constant. The *expectations effect* is the effect of a change in x_1 on profits caused by a change in expectations consistent with a fulfilled-expectations equilibrium of the continuation game. The direct and strategic effects are well-known in the strategic-management literature (see Besanko, Dranove, Shanley, and Schaefer, 2012, for example). The study of the expectations effect is a contribution of the present paper.

Let $\pi(s_i, \phi_i, x_i) = (1 - \phi_i)\, x_i\, s_i - c\, x_i^2/2$ be the profit of firm i for given s_i, ϕ_i, and x_i. Let $\hat{\phi}(x_i, x_j, s_i^e)$ and $\hat{s}(x_i, x_j, s_i^e)$ be the equilibrium openness degree and market share of firm i in the third stage of the game, given investments x_1, x_2 and expectations s_i^e. Let $s^*(x_i, x_j)$ be an equilibrium market share for firm i consistent with fulfilled expectations, and note that, given x_1 and x_2, $s^*(x_i, x_j)$ is one of the solutions to $s^* = \hat{s}(x_i, x_j, s^*)$. Given that there may be more than one solution to this equation, I select the equilibrium closest to an equal distribution of sales. Let $\phi^*(x_1, x_2) = \hat{\phi}(x_i, x_j, s^*(x_i, x_j))$ be the equilibrium openness degree of firm i in such fulfilled-expectations equilibrium. I denote partial derivatives with subindices, e.g., $\pi_{s_i}(s_i, \phi_i, x_i) = \partial\pi(s_i, \phi_i, x_i)/\partial s_i$, $\hat{\phi}_{x_j}(x_i, x_j, s_i^e) = \partial\hat{\phi}(x_i, x_j, s_i^e)/\partial x_j$, and $s_{x_i}^*(x_i, x_j) = \partial s^*(x_i, x_j)/\partial x_i$.

The total effect of a change in x_1 is

$$\begin{aligned} TE &= \pi_{s_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1)\, s_{x_1}^*(x_1, x_2) \\ &\quad + \pi_{\phi_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1)\, \phi_{x_1}^*(x_1, x_2) \\ &\quad + \pi_{x_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1), \end{aligned}$$

the direct effect is

$$\begin{aligned} DE &= \pi_{s_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1)\, s_{x_1}^d(\phi^*(x_1, x_2), \phi^*(x_2, x_1), x_1, x_2, s^*(x_1, x_2)) \\ &\quad + \pi_{x_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1), \end{aligned}$$

the strategic effect is

$$\begin{aligned} SE &= \pi_{s_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1)\, \hat{s}_{x_1}(x_1, x_2, s^*(x_1, x_2)) \\ &\quad + \pi_{\phi_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1)\, \hat{\phi}_{x_1}(x_1, x_2, s^*(x_1, x_2)) \\ &\quad + \pi_{x_1}(s^*(x_1, x_2), \phi^*(x_1, x_2), x_1) - DE, \end{aligned}$$

and the expectations effect is

$$EE = TE - DE - SE.$$

After some straightforward calculations, it can be shown that the effects, evaluated in a symmetric equilibrium ($x_1^* = x_2^* = x^*$), are

$$DE = \frac{1}{2} - c\,x^*, \tag{5}$$

$$SE = -\frac{1}{6}, \tag{6}$$

$$EE = \frac{\sigma\,(4\,(2+\sigma)\,x^*\,f(0) - 3)}{6\,(2+\sigma)\,(3 - 2\,\sigma\,x^*\,f(0))}, \tag{7}$$

$$TE = \frac{4 + \sigma - 2\,c\,(2+\sigma)\,x^*\,(3 - 2\,\sigma\,x^*\,f(0))}{2\,(2+\sigma)\,(3 - 2\,\sigma\,x^*\,f(0))}. \tag{8}$$

The direct effect can be positive or negative, depending on the size of the investment. The strategic effect is always negative: third-stage actions (openness degrees) are strategic complements, and a larger investment in product quality leads to more aggressive competition (i.e., it acts as a tough commitment), which leads to lower profits (Fudenberg and Tirole, 1984; Besanko et al., 2012).

More surprisingly, the expectations effect may be positive or negative. An increase in a firm's investment leads to an increase in the expected network size of that firm (Proposition 5) which leads to an increase in the demand of that firm. This effect is always positive. However, the increase in expectations also has an effect on strategic interaction in the third stage of the game, and this effect is negative (for the same reason that the strategic effect is negative). As a consequence, the expectations effect may have either sign. If the expectations effect is negative, it means that *firms will invest less in product development* than what they would invest if investments did not affect users' expectations. The following proposition formalizes these claims.

Proposition 6 (Endogenous quality investment and expectations effect)**.** *A symmetric equilibrium exists and is unique. Equilibrium investment is*

$$x^* = \frac{4+\sigma}{3\,c\,(2+\sigma) + \sqrt{c\,(2+\sigma)\,(9\,c\,(2+\sigma) - 4\,f\,\sigma\,(4+\sigma))}},$$

which is decreasing in c and increasing in $f(0)$. If $c > \frac{\sigma\,(2+\sigma)}{2\,(1+\sigma)}\,f(0)$ $\left(c < \frac{\sigma\,(2+\sigma)}{2\,(1+\sigma)}\,f(0)\right)$, the direct effect is positive (negative). If $c < \frac{8}{9}\,f(0)$ $\left(c > \frac{8}{9}\,f(0)\right)$, then there exists $\overline{\sigma} > 0$ such that, for $\sigma < \overline{\sigma}$, x^* is increasing (decreasing) in σ and the expectations effect is positive (negative).

A surprising result of proposition 6 is that a larger user-innovation parameter may lower equilibrium investments. This effect arises because firms take into account the expectations effect when choosing x^*. To see this, note that if firms did not take into account the expectations effect they would choose x^* to make $DE + SE = \frac{1}{3} - c\,x^* = 0$, in which case, x^* would be unresponsive to changes in σ. On the contrary, firms choose x^* to make $DE + SE + EE = 0$. Thus, the effect of a change in σ on x^* depends on whether a larger σ increases or decreases the expectations effect. The proof of Proposition 6 confirms this intuition: for small σ, the derivative of EE with respect to σ has the same sign than derivative of x^* with respect to σ.

9. Multi-firm competition and entry

In this section, I extend the basic model to introduce competition between $n \geq 2$ firms. To obtain closed form solutions for demands, I assume that the idiosyncratic taste shocks ε_{ik} are distributed according to a double exponential distribution, which leads to logit demands (Anderson, de Palma, and Thisse, 1992). The cumulative distribution is

$$F(\omega) = \exp(-\exp(-\nu - \omega/\mu)),$$

where ν is Euler's constant ($\nu \approx 0.5772$) and μ is a positive constant measuring users' taste heterogeneity. When $n = 2$, the double exponential distribution implies that the difference $\varepsilon_{1k} - \varepsilon_{2k}$ is distributed according to a logistic distribution, so the equilibrium is the one obtained in Section 5.

Assume that $x_i = x$ for all i. In equilibrium, firms set $p_i = (1 - \phi_i)\,x$ and the demand for product i is

$$s^d(\phi_i, \phi_{-i}, x_i, \mathbf{x}_{-i}, s_i^e, \mathbf{s}_{-i}^e) = \frac{\exp((1 + \sigma\, s_i^e)\,\phi_i\, x/\mu)}{\sum_{j=1}^n \exp((1 + \sigma\, s_j^e)\,\phi_j\, x/\mu)},$$

where the $-i$ subindex represents firms other than i (e.g., ϕ_{-i} is a vector containing the openness degrees of all firms except i). Proposition 7 characterizes the symmetric equilibrium of the model.

Proposition 7 (Multi-firm competition)**.** *A symmetric equilibrium ($s_i = 1/n$ for all i) exists and is unique. The equilibrium openness degree in the symmetric equilibrium is*

$$\phi^* = 1 - \frac{\mu\, n^2}{(n + \sigma)\, x\, (n - 1)},$$

which is increasing in n. Equilibrium prices and profits are decreasing in n.

Proposition 7 has two main implications. First, it shows that previous results in the duopoly model extend to competition between more than two firms. Second, it shows that entry leads to an increase in openness. Thus, we should expect firms to be more open in industries with smaller entry costs.

10. Endogenous user investments

In this section, I extend the model to allow for endogenous user investments. The main results of the paper still hold, but we can study the effects of increasing the importance of user investments in utility on equilibrium openness decisions.

Return to the duopoly framework of section 3 and assume $x_1 = x_2 = x$. User k's indirect utility for consuming firm i's proprietary product is

$$v_{ik}^p = q_i^p - p_i + \omega \left(\sigma \, \phi_i \, x \, y_k - \frac{1}{2} \, {y_k}^2 \right) + \varepsilon_{ik},$$

where y_k is user k's investment, $\sigma \, \phi_i \, x \, y_k$ is an idiosyncratic benefit from investment, ${y_k}^2/2$ is user k's investment cost, and $\omega > 0$ is a parameter measuring the importance of idiosyncratic investments in utility. User k's indirect utility for consuming firm i's open product is

$$v_{ik}^o = q_i^o + \omega \left(\sigma \, \phi_i \, x \, y_k - \frac{1}{2} \, {y_k}^2 \right) + \varepsilon_{ik}.$$

Total user innovation on the products of firm i is $z_i = \int_{K_i} y_k \, dk$, where K_i is the set of users choosing firm i' products. In equilibrium, firms set $p_i = (1 - \phi_i) \, x$, and users consume one of the two proprietary goods offered by the firms.

If user k chooses a product of firm i, her optimal investment is

$${y_k}^* = \sigma \, \phi_i \, x.$$

which means that users expect $z_i^* = \sigma \, \phi_i \, x \, s_i^e$. The utility of choosing the proprietary product of firm i is

$$v_{ik}^p = (1 + \sigma s_i^e) \, \phi_i \, x_i + \omega \, \frac{1}{2} \, (\sigma \, \phi_i \, x)^2 + \varepsilon_{ik},$$

and the demand of firm i is

$$s^d(\phi_i, \phi_j, s_i^e) = F\left((1 + \sigma s_i^e) \, \phi_i \, x + \frac{\omega}{2} \, (\sigma \, \phi_i \, x)^2 - (1 + \sigma(1 - s_i^e)) \, \phi_j \, x - \frac{\omega}{2} \left(\sigma \, \phi_j \, x \right)^2 \right).$$

It is straightforward to see that all previous results hold when ω is close to 0. The following proposition shows the equilibrium and comparative statics as ω grows.

Proposition 8 (Endogenous user investment)**.** *A fulfilled-expectations equilibrium exists. The number of equilibria is finite and odd. A symmetric equilibrium ($s_1^* = s_2^* = 1/2$) exists* *and is unique. Equilibrium openness and individual user investments are increasing in ω for small ω and $s_i^* \geq 1/2$, but may be increasing or decreasing in ω for small ω and $s_i^* < 1/2$.*

The lemma shows that the effect of an increase in the importance of idiosyncratic investments depends on equilibrium market shares. For small ω, in the symmetric equilibrium, openness increases with ω, as well as individual user investments, whereas prices and firm profits decrease. In an asymmetric equilibrium, the sign of the effects remains unchanged for the larger firm, but they may be reversed for the smaller firm. Thus, small firms may decrease their openness degree as user investments become more important.

11. Conclusion

I study firm's openness and product development decisions in competitive environments with open and user innovation. I ask the following questions: What are the determinants of the optimal openness degree? Should large or small firms be more open? How should a firm respond to increased openness by a rival? Should a firm try to be a first mover? Should firms pursue greater compatibility and spillovers? How does the collaboration/appropriability trade off affect the incentives to invest in product development? What are the effects of firm entry on openness and profits?

I present several testable results. First, I show that product-market competition, the intensity of user innovation (number of contributions per user), and the size of the knowledge stock have a positive effect on openness decisions, and that firm size is negatively related with the openness degree. This last result has direct managerial and empirical implications, and can be tested with data on open-source projects, measuring openness as the proportion of (compiled or source) code released under an open-source license. Firm size can be measured by market share or by the number of employees. A good precedent for this type of research is Bonaccorsi et al. (2006). These authors present a survey of Italian software firms and show that most firms combine proprietary and OS products and receive revenues from traditional license fees as well as from open-source related services. They also examine the determinants of the degree of openness, and find that size (measured as the number of employees) is negatively correlated with openness. This paper provides a potential explanation.

Second, I show that openness decisions are strategic complements, which explains why firms usually react to higher openness by rivals by becoming more open, as in the cases of IBM versus JBoss and Facebook versus OpenSocial. This implication can also be tested empirically, which is an interesting direction for further research.

Third, I show that the game exhibits second-mover advantages. There are two reasons for this result: (i) actions are strategic complements (this effect is well known in the literature, see Gal-Or, 1985, for example), and (ii) moving first has a negative effect on expectations. Users know that the first mover has incentives to choose a smaller openness degree than its rival, and thus, adjust the expectations over the market share of the first mover downwards. It may even be the case that a firm looses if it becomes a first mover. This result would not be possible in a market without network effects, and is an important contribution of this paper. From a managerial point of view, the result shows that it is important to take users' expectations into account when determining the desirability of making strategic commitments.

Finally, I show that product-development incentives are affected by the expectation-formation mechanism. As is well known, investing in product quality has a direct and a strategic effect on profits (Fudenberg and Tirole, 1984; Besanko et al., 2012). I show that investments may have an additional effect through user's expectations, which affects product-development decisions. Surprisingly, this *expectations effect may be positive or negative.* For example, I find that when product-development is costly relative to the intensity of product-market competition, a larger investment has a negative expectations effect on profits. As a result, firms will invest less in product development than what they would invest if investments did not affect users' expectations. Moreover, I find that in this case, equilibrium investment decreases as the number of contributions per user increases.

Appendix: Proofs

Proof of Proposition 1. The first order condition is

$$-x\, s_i^d + (1-\phi_i)\, x\, f\Big((1+\sigma s_i^e)\phi_i x - (1+\sigma s_j^e)\phi_j x\Big)\, (1+\sigma s_i^e)\, x = 0.$$

The assumption of an increasing hazard ratio guarantees that the second order condition holds. Fulfilled expectations imply $s_i = s_i^e$. Rearranging the first order condition, I obtain

$$(1+\sigma s_i)\, \phi_i\, x = (1+\sigma s_i)\, x - \frac{s_i}{f\Big((1+\sigma s_i)\phi_i x - (1+\sigma s_j)\phi_j x\Big)}.$$

By symmetry of f, $f\Big((1+\sigma s_i)\phi_i x - (1+\sigma s_j)\phi_j x\Big) = f\Big((1+\sigma s_j)\phi_j x - (1+\sigma s_i)\phi_i x\Big)$. Substituting this expression into demands and first-order conditions, I obtain the equilibrium market shares and openness degrees. Let $H(s) = s - F\left((1-2s)\left(\frac{1}{f(F^{-1}(s))} - \sigma x\right)\right)$. There exists an equilibrium for any $s \in [0,1]$ such that $H(s) = 0$. It is straightforward to see that

$H(s)$ is continuous, $H(0) < 0$ and $H(1) > 0$. Thus, by the intermediate value theorem, there must exist a finite and odd number of equilibria. ■

Proof of Proposition 2. By Proposition 1, $H(1/2) = 0$. Thus, if $H'(1/2) < 0$ there exists at least two asymmetric equilibria, one with $s^* < 1/2$ and one with $s^* > 1/2$. It is straightforward to see that $\sigma\, x\, f(0) > 3/2$ guarantees $H'(1/2) < 0$. To see that the larger firm is less open than its rival, note that the ratio

$$\frac{1-\phi_i^*}{1-\phi_j^*} = \frac{\frac{s_i^*}{(1+\sigma s_i^*)x f(F^{-1}(s_i^*))}}{\frac{s_j^*}{(1+\sigma s_j^*)x f(F^{-1}(s_j^*))}} = \frac{\frac{1}{s_j^*}+\sigma}{\frac{1}{s_i^*}+\sigma}$$

is larger than 1 if $s_i^* > 1/2 > s_j^*$. Thus, $\phi_i^* < \phi_j^*$ if i has a larger market share than j. Finally, suppose that $\sigma\, x\, f(0) \leq 1$, which implies $\frac{1}{f(0)} - \sigma x \geq 0$. At $s = 1/2$, $H(s) = 0$. It is straightforward to see that $F\left((1-2s)\left(\frac{1}{f(F^{-1}(s))} - \sigma x\right)\right) < F(0)$ for any $s > 1/2$. To see this, note that $1-2s < 0$ for $s > 1/2$, and $\frac{1}{f(F^{-1}(s))} - \sigma x > 0$, given that $f(\omega)$ has its maximum at $\omega = 0$ and $\frac{1}{f(0)} - \sigma x \geq 0$. This result implies that $H(s) > 0$ for any $s > 1/2$. Similar arguments imply that $H(s) < 0$ for any $s < 1/2$. ■

Proof of Proposition 3. Let $\phi_2(\phi_1)$ be firm 2's best response given firm 1's openness degree, and let $s_1(\phi_1)$ and $s_2(\phi_1)$ be the equilibrium market shares of firms 1 and 2 given ϕ_1 and $\phi_2(\phi_1)$. Noting that $s_2(\phi_1) = 1 - s_1(\phi_1)$ and that f is symmetric, from Proposition 1 it follows that

$$\phi_2(\phi_1) \;=\; 1 - \frac{1-s_1(\phi_1)}{(1+\sigma s_2^e)\, x\, f(F^{-1}(s_1(\phi_1)))}, \tag{9}$$

$$s_1(\phi_1) \;=\; (1+\sigma s_1^e)\,\phi_1\, x - (1+\sigma s_2^e)\, x + \frac{1-s_1(\phi_1)}{f(F^{-1}(s_1(\phi_1)))}. \tag{10}$$

In the first stage, firm 1 chooses ϕ_1 to maximize $(1-\phi_1)\, x\, s_1(\phi_1)$. The first-order condition is

$$-x\, s_1(\phi_1) + (1-\phi_1)\, x\, s_1'(\phi_1) = 0.$$

Differentiating (10) with respect to ϕ_1 and solving for $s_1'(\phi_1)$ I obtain

$$s_1'(\phi_1) = \frac{(1+\sigma s_1^e)\, x\, f(F^{-1}(s_1(\phi_1)))}{1+g(F^{-1}(s_1(\phi_1)))}. \tag{11}$$

where $g(\omega) = h'(\omega)/h(\omega)^2 > 0$. Introducing this expression in (11), imposing $s_1^e = s_1^*$, and solving for ϕ_1, I obtain that the equilibrium openness degree is

$$\phi_1^* = 1 - \frac{s_1^*}{(1+\sigma s_1^*)\, x\, f(F^{-1}(s_1^*)}\,(1+g(F^{-1}(s_1^*)),$$

where the equilibrium market share is implicitly given by

$$s_1^* \;=\; F\left((1-2s_1^*)\left(\frac{1}{f(F^{-1}(s_1^*))}-\sigma x\right)-\frac{s_1^*}{f(F^{-1}(s_1^*))}\,g(F^{-1}(s_1^*))\right),$$

Let $H(s) = s - F\left((1-2s)\left(\frac{1}{f(F^{-1}(s))}-\sigma x\right)-\frac{s}{f(F^{-1}(s))}\,g(F^{-1}(s))\right)$. An equilibrium exists for any s such that $H(s) = 0$. It is straightforward to see that $H(0) < 0$ and $H(1) > 0$. Thus, by continuity and the intermediate value theorem, there must exist a finite and odd number of values of s for which $H(s) = 0$. Also, note that $H(1/2) > 0$, which means that there must exist some $s < 1/2$ for which $H(s) = 0$. To see that if $\sigma\, x\, f(0) \le 1$, all equilibria have $s_1^* < 1/2 < s_2^*$, note that this implies that $(1-2s)\left(\frac{1}{f(F^{-1}(s))}-\sigma x\right) < 0$ for any $s > 1/2$, and thus $H(s) > 0$ for any $s > 1/2$.

To see that actions are strategic complements, note that (9) can be written as

$$\phi_2(\phi_1) = 1 - \frac{1}{(1+\sigma s_2^e)\, x\, h(F^{-1}(s_1(\phi_1)))}, \tag{12}$$

Given that $s_1'(\phi_1) > 0$ by (11) and that $h' > 0$, actions are strategic complements. Applying standard results from Gal-Or (1985), it follows that there are second-mover advantages, and that for any $s_1^* < 1/2 < s_2^*$, firm 1 has lower profits than firm 2. ■

Proof of Proposition 4. The proof for most results follows the same steps as the proofs of Propositions 1 and 2. In the symmetric equilibrium, profit is $\pi^* = (2\,(2+\sigma(1-\beta))\, f(0))^{-1}$, which is increasing in β. ■

Proof of Proposition 5. Follows the same steps as the proof of Proposition 3. ■

Proof of Proposition 6. Equilibrium investment is obtained by making $TE = 0$ and solving for x^*. Comparative statics with respect to c and $f(0)$ follow from direct inspection of x^*. Substituting x^* in DE, it is straightforward to show that $DE > (<)\ 0$ if and only if $c > (<)\ \frac{\sigma\,(2+\sigma)}{2\,(1+\sigma)}\, f(0)$. The derivative of x^* with respect to σ, evaluated at $\sigma = 0$ is $(8\, f(0) - 9\, c)/(108\, c^2)$. Substituting x^* in 7, and evaluating at $\sigma = 0$, I obtain that $EE = 0$. Applying L'Hopital's rule, it is straightforward to show that the derivative of EE with respect to σ converges to $(8\, f(0) - 9\, c)/(108\, c)$ as $\sigma \to 0$. Results follow from continuity of x^* and EE with respect to σ. ■

Proof of Proposition 7. Follows the same steps as the proofs of Prop. 1 and 2. ■

Proof of Proposition 8. The first order condition is

$$-x\, s_i^d + (1-\phi_i)\, x\, f\ \left((1+\sigma s_i^e)\, x + \omega\, (\sigma\, x)^2\, \phi_i\right) = 0.$$

where

$$f = f\left((1+\sigma s_i^e)\,\phi_i\,x + \frac{\omega}{2}\,(\sigma\,\phi_i\,x)^2 - (1+\sigma(1-s_i^e))\,\phi_j\,x - \frac{\omega}{2}\left(\sigma\,\phi_j\,x\right)^2\right).$$

The proofs for the existence and number of equilibria, and for the existence and uniqueness of the symmetric equilibrium follow the same steps as the proof of Proposition 1. By the implicit function theorem, the sign of $\partial\phi_i^*/\partial\omega$ depends on the sign of $\partial^2\pi_i/\partial\phi_i\,\partial\omega$. Differentiating and taking the limit as $\omega \to 0$, I obtain

$$\lim_{\omega\to 0}\frac{\partial^2\pi_i}{\partial\phi_i\,\partial\omega} = \frac{1}{2}\sigma^2\,x^3\,\left(\phi_i\,(2-3\phi_i)+\phi_j^2\right)\,f + \frac{1}{2}\sigma^2\,x^3\,(1-\phi_i)\,x\,(1+\sigma s_i)\,(\phi_i^2-\phi_j^2)\,f',$$

where

$$f' = f'\left((1+\sigma s_i^e)\,\phi_i\,x - (1+\sigma(1-s_i^e))\,\phi_j\,x\right).$$

By previous results, we know that if $s_i^* > 1/2$, then $\phi_i < \phi_j$ and $f' < 0$. Likewise, if $s_i^* < 1/2$, then $\phi_i > \phi_j$ and $f' > 0$. Therefore, the second term is positive if $s_i^* \neq 1/2$, and is equal to zero if $s_i^* = 1/2$. It is straightforward so show that $\phi_i\,(2-3\phi_i)+\phi_j^2$ is positive if $\phi_i < \phi_j$ and is negative if $\phi_i > \phi_j$. The result follows by noting that the second term can be made as small as wanted by lowering f' (f converges to the cdf of a uniform distribution as $f' \to 0$). ■

References

Allen, R. C. (1983): "Collective invention," *Journal of Economic Behavior and Organization*, 4, 1–24.

Almirall, E. and R. Casadesus-Masanell (2010): "Open versus closed innovation: A model of discovery and divergence," *Academy of Management Review*, 35, 27–47.

Anderson, S., A. de Palma, and J. Thisse (1992): *Discrete Choice Theory of Product Differentiation*, MIT Press, Cambridge, MA.

Anton, J. J. and D. A. Yao (1994): "Expropriation and inventions: Appropriable rents in the absence of property rights," *American Economic Review*, 190–209.

Arrow, K. J. (1962): "Economic Welfare and the Allocation of Resources for Invention," in *The Rate and Direction of Inventive Activity*, Princeton University Press, Princeton, NJ.

Athey, S. and G. Ellison (2010): "Dynamics of open source movements," CESifo Working Paper Series 3215, CESifo GmbH.

Baldwin, C. Y. and K. B. Clark (2006): "The Architecture of Participation: Does Code Architecture Mitigate Free Riding in the Open Source Development Model?" *Management Science*, 52, 1116–1127.

Besanko, D., D. Dranove, M. Shanley, and S. Schaefer (2012): *Economics of Strategy*, John Wiley & Sons, Hoboken, NJ.

Bessen, J. (2006): "Open Source Software: Free Provision of Complex Public Goods," in *The Economics of Open Source Software Development*, ed. by J. Bitzer and P. Schröder, Elsevier, Amsterdam, Netherlands.

Bonaccorsi, A., S. Giannangeli, and C. Rossi (2006): "Entry strategies under competing standards: Hybrid business models in the open source software industry," *Management Science*, 52, 1085–1098.

Bonaccorsi, A. and C. Rossi (2004): "Altruistic individuals, selfish firms? The structure of motivation in open source software," *First Monday*, 9.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985): "Multimarket oligopoly: strategic substitutes and complements," *Journal of Political Economy*, 93, 488–511.

Casadesus-Masanell, R. and P. Ghemawat (2006): "Dynamic mixed duopoly: a model motivated by Linux vs. Windows," *Management Science*, 52, 1072–1084.

Casadesus-Masanell, R. and G. Llanes (2011): "Mixed source," *Management Science*, 57, 1212–1230.

Chesbrough, H. (2003): *Open Innovation: The New Imperative for Creating and Profiting from Technology*, Harvard Business School Press, Boston, MA.

Chesbrough, H. and A. K. Crowther (2006): "Beyond high tech: early adopters of open innovation in other industries," *R&d Management*, 36, 229–236.

Chesbrough, H. W. and M. M. Appleyard (2007): "Open innovation and strategy," *California Management Review*, 50, 57–76.

Christensen, J. F., M. H. Olesen, and J. S. Kjær (2005): "The industrial dynamics of Open Innovation—Evidence from the transformation of consumer electronics," *Research policy*, 34, 1533–1549.

Fershtman, C. and N. Gandal (2011): "Direct and indirect knowledge spillovers: the "social network" of open-source projects," *RAND Journal of Economics*, 42, 70–91.

Fudenberg, D. and J. Tirole (1984): "The fat-cat effect, the puppy-dog ploy, and the lean and hungry look," *American Economic Review*, 74, 361–366.

Gal-Or, E. (1985): "First mover and second mover advantages," *International Economic Review*, 26, 649–653.

Gambardella, A. and C. Panico (2014): "On the management of open innovation," *Research Policy*, 43, 903–913.

Harhoff, D., J. Henkel, and E. von Hippel (2003): "Profiting from voluntary information spillovers: how users benefit by freely revealing their innovations," *Research* *policy*, 32, 1753–1769.

Hellmann, T. and E. Perotti (2011): "The circulation of ideas in firms and markets," *Management Science*, 57, 1813–1826.

Henkel, J. (2006): "Selective revealing in open innovation processes: The case of embedded Linux," *Research Policy*, 35, 953–969.

Henkel, J., S. Schöberl, and O. Alexy (2014): "The emergence of openness: How and why firms adopt selective revealing in open innovation," *Research Policy*, 43, 879–890.

Johnson, J. (2002): "Open source software: private provision of a public good," *Journal of Economics and Management Strategy*, 11, 637–662.

Katz, M. L. and C. Shapiro (1985): "Network externalities, competition, and compatibility," *American Economic Review*, 75, 424–440.

Lakhani, K. and R. Wolf (2005): "Why Hackers Do What They Do: Understanding Motivation and Effort in Free/Open Source Software Projects," in *Perspectives on Free and Open Source Software*, ed. by J. Feller, B. Fitzgerald, S. Hissam, and K. R. Lakhani, MIT Press, 3–22.

Laursen, K. and A. Salter (2006): "Open for innovation: the role of openness in explaining innovation performance among UK manufacturing firms," *Strategic Management Journal*, 27, 131–150.

Llanes, G. and R. de Elejalde (2013): "Industry equilibrium with open-source and proprietary firms," *International Journal of Industrial Organization*, 31, 36–49.

MacCormack, A., J. Rusnak, and C. Y. Baldwin (2006): "Exploring the Structure of Complex Software Designs: An Empirical Study of Open Source and Proprietary Code," *Management Science*, 52, 1015–1030.

Morrison, P. D., J. H. Roberts, and E. von Hippel (2000): "Determinants of user innovation and innovation sharing in a local market," *Management Science*, 46, 1513–1527.

Niedermayer, A. (2013): "On platforms, incomplete contracts, and open source software," *International Journal of Industrial Organization*, 31, 714–722.

Oh, W. and S. Jeon (2007): "Membership herding and network stability in the open source community: The Ising perspective," *Management Science*, 53, 1086–1101.

Polanski, A. (2007): "Is the General Public License a rational choice?" *Journal of Industrial Economics*, 55, 691–714.

Raymond, E. (1999): *The Cathedral and the Bazaar: Musings on Linux and Open Source from an Accidental Revolutionary*, O'Reilly and Associates, Sebastopol, CA.

Reisinger, M., L. Ressner, R. Schmidtke, and T. P. Thomes (2014): "Crowding-in of complementary contributions to public goods: Firm investment into open source software," *Journal of Economic Behavior & Organization*, 106, 78–94.

Roberts, J. A., I.-H. Hann, and S. A. Slaughter (2006): "Understanding the motivations, participation, and performance of open source software developers: A longitudinal study of the Apache projects," *Management Science*, 52, 984–999.

Rysman, M. and T. Simcoe (2008): "Patents and the performance of voluntary standard-setting organizations," *Management Science*, 54, 1920–1934.

Shah, S. K. (2006): "Motivation, governance, and the viability of hybrid forms in open source software development," *Management Science*, 52, 1000–1014.

Stallman, R. (2002): *Free Software, Free Society: Selected Essays of Richard M. Stallman*, Free Software Foundation, Boston, MA.

von Hippel, E. (1988): *The Sources of Innovation*, Oxford University Press, New York, NY.

——— (2005): *Democratizing Innovation*, MIT Press, Cambridge, MA.

West, J. (2003): "How open is open enough?: Melding proprietary and open source platform strategies," *Research Policy*, 32, 1259–1285.

West, J., A. Salter, W. Vanhaverbeke, and H. Chesbrough (2014): "Open innovation: the next decade," *Research Policy*, 43, 805–811.