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Anticommons and Optimal Patent Policy in a Model of Sequential Innovation*

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Abstract

We present a model of sequential innovation in which innovators use several research inputs to invent new goods. We extend work by Shapiro (2001) and Lerner and Tirole (2004) by studying the effects of increases in the number of patented research inputs on innovation incentives and optimal patent policy. We consider not only the effects on the incentives to invent final goods, but also on the incentives to invent research inputs (ex-ante effect). We find increasing complexity has a negative effect on innovation activity in the final goods sector when research inputs are complements. Either limiting market power through weaker patents or reducing the lack of coordination through patent pools may solve this problem. We also find the optimal patent breadth and show it is increasing in the elasticity of substitution between the inputs used in research and decreasing (increasing) in the complexity of the R&D process when research inputs are complements (substitutes).

KEYWORDS: anticommons, sequential innovation, patent policy, intellectual property rights, complementary monopoly, patent pools, returns to specialization

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1 Introduction

Innovation is cumulative by nature. By and large, innovators build upon the work of previous inventors, either because they use the results of previous research as inputs in the R&D process, or simply because the new product embodies components that are previous inventions themselves. Depending on the intellectual property rights regime in place, patents might protect previous inventions, in which case the innovator will have to pay licensing fees for the use of these technologies.

If innovators need to access a large number of previous inventions, they may face a *patent thicket* (Shapiro, 2001), and become trapped in a *tragedy of the anticommons* (Heller, 1998). A patent thicket is "a dense web of overlapping intellectual property rights that a company must hack its way through in order to actually commercialize new technology" (Shapiro, 2001). Patent thickets are pervasive in hi-tech industries such as software, hardware, biotechnology, and electronics. As examples, in the 1980s, IBM accused Sun Microsystems of infringing on some of its 10,000 software patents (Reback, 2002), development of golden rice required the use of around 40 patented products and processes (Graff, Cullen, Bradford, Zilberman, and Bennett, 2003), and 39 patent families are "potentially relevant in developing a malaria vaccine from [the protein] MSP-1" (Commision on Intellectual Property Rights, 2002).

Anticommons arise when multiple owners have exclusion rights over the use of a common resource, causing its inefficient under-utilization. The intuition is similar to that of the complementary monopoly problem (Cournot, 1838, Sonnenschein, 1968, Bergstrom, 1978). Imagine two inputs, A and B, are necessary for the production of a final good with downward-sloping demand. Increasing the price of A increases the marginal cost of producing the final good, reducing its demand and, consequently, the demand for input B. If two different firms sell A and B, the cross-price effect is neglected and the total cost of producing the final good increases, in comparison with the case in which only one firm sets the price of the two inputs.

Shapiro (2001) proposes the creation of patent pools to avoid the complementary monopoly problem patent thickets cause. A patent pool is a cooperative agreement among patent holders, through which they agree on the licensing terms of a subset of their patents. In the previous example, if a single monopolist sells A and B (or if two firms set their prices cooperatively, as in a patent pool), the cross-price effect is taken into account, which leads to a lower price for the inputs.

Lerner and Tirole (2004) generalize Shapiro's analysis for varying degrees of substitution between the extremes of perfect complements and perfect substitutes. An increase in the degree of substitution between the patented inventions reduces the market power of the patent holders, which leads to lower prices. If substitutability is high enough, cooperative price setting will lead to higher prices than uncoordinated price setting. Therefore, patent pools are socially desirable only when they are composed of complementary patents. However, Lerner and Tirole focus on determining the welfare effects of patent pools and do not study how increases in the size of the thicket affect innovation activity and optimal patent policy. Moreover, in Lerner and Tirole, inputs have already been invented when the new invention is being considered. Therefore, the authors focus on the ex-post rather than ex-ante effect of patent pools.

In this paper, we extend the work by Shapiro (2001) and Lerner and Tirole (2004) in three directions: (i) we study what happens with innovation activity as the number of patented inputs potentially involved in the innovation process increases; (ii) we determine the optimal patent policy as a function of the number of research inputs; and (iii) we consider not only the incentives to create the new final good, but also the incentives to create the research inputs (ex-ante effect). By doing so, we connect the literatures of complementary monopoly and patent pools to the literature of sequential innovation.

Two key insights of the paper are: (i) increasing complexity may have a negative effect on innovation activity when research inputs are complementary, and (ii) either limiting market power through weaker patents or reducing the lack of coordination through the creation of patent pools may solve this problem.

Our paper is also related to the literature of sequential innovation (Scotchmer, 1991, Green and Scotchmer, 1995, Chang, 1995, Scotchmer, 1996, Hopenhayn, Llobet, and Mitchell, 2006), which focuses primarily on the optimal division of profits between sequential innovators. Generally, in these models, two innovations must be introduced sequentially, and the objective is to find the patent policy that maximizes the incentives to invest in both innovations. In this paper, we generalize these models by assuming the innovation is based on several previous innovations. In this sense, an important precedent of our paper is Boldrin and Levine (2005a), who were the first to apply the structure of complementary monopoly to a model of sequential innovation. Here, we extend their analysis to an oligopoly framework with any degree of substitutability.

Similarly, our paper is related to Llanes and Trento (2010), which presents a dynamic model in which a sequence of innovations may be introduced by different innovators, and where each innovation builds on several prior inventions. Llanes and Trento (2010) studies three scenarios –patents, no patents, and patent pools– and shows that innovation is suboptimal in all of them. However, the primary concern of that paper is to study how dynamic incentives affect our thinking of the complementary monopoly problem, for which it focuses on the perfect complements case (which Cournot 1838, Shapiro 2001, and Boldrin and Levine 2005a analyzed in a static setting). The focus of this current paper is different, precisely because we want to study the interactions between patent policy and differing degrees of

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substitution/complementarity, for which we have to move beyond the perfect complements case. For example, we find the optimal patent breadth is increasing in the elasticity of substitution between the inputs used in research, and is decreasing (increasing) in the complexity of the R&D process when research inputs are complements (substitutes), which would not have been possible without generalizing the degree of substitution.

2 The model

We study a model in which a continuum of potential innovators may use n inputs (or a subset of them) in R&D to invent new goods. The value of the new goods, v, is heterogeneous across the innovators, and each innovator knows the value of her innovation before performing R&D. For simplicity, and following Lerner and Tirole (2004), we assume innovators do not compete in the final goods market; that is, each innovator sells her new good in a different market.

The timing of the game is as follows: (i) input producers set the price for their research inputs; and (ii) innovators observe input prices, choose the combination of inputs that minimize the cost of R&D, and decide whether to innovate.

Given that research inputs are imperfect substitutes for each other, the factor market is a differentiated goods oligopoly. In this section, we study what happens with innovation activity when the number of research inputs increases, assuming full patent protection. In section 4, we allow for partial patent protection and determine the optimal patent policy as a function of the number of research inputs.

2.1 Technology

Each input is produced with a constant marginal cost of $\varepsilon > 0$. The resources used to produce inputs are sold in a competitive market, so the private and social costs of producing inputs coincide. All innovators share the same R&D technology,

$$y = A(n) \left(\sum_{i=1}^{n} \omega_i \, x_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{1}$$

where y is a measure of the R&D effort, x_i is the amount of input i used in R&D, $\omega_i = 1/n$ is the weight of input i in the research technology, n is the number of available inputs, $\sigma \in [0,\infty)$ is the elasticity of substitution between the inputs, and $A(n)$ is a scale parameter.

Let $a(n) = A(n)/n$, where $a(n)$ measures the returns to specialization.[1] If $a'(n) = 0$, there are no returns to specialization; if $a'(n) > 0$, there are positive returns to specialization; and if $a'(n) < 0$, there are negative returns to specialization. We begin our analysis by focusing on the case without returns from specialization ($a(n) = 1$). In section 3.5 we analyze how results change when $a'(n) \neq 0$.

Innovators face an indivisibility problem. Inventing a new good requires a minimum amount of R&D effort. When the R&D effort is below the threshold, no innovation occurs. Without loss of generality, we can set the threshold at 1 so the indicator function for the innovation is

$$I = \begin{cases} 1 & \text{if } y \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

In section 2.3 we discuss in greater detail the intuition behind our innovation technology. Before doing so, we continue with the description of the model.

The complexity of the innovation process is measured by the number of inputs used in R&D. More complex technologies use a larger number of research inputs. Given the functional form of our research technology (CES), when $\sigma \in [0,1]$ innovators must use all inputs in a positive quantity for $y > 0$. When $\sigma > 1$, on the other hand, innovators *could* use a proper subset of the n inputs and still invent a new good. However, in section 3, we show that in equilibrium all inputs are used in a positive quantity.

The assumption of no returns to specialization guarantees the *social* cost of performing the innovations does not change as technologies become more complex. In other words, increases in n produce no technological advantage or disadvantage. This assumption is important because it allows us to study how the *private* cost of innovation diverges from the *social* cost of innovation as n increases.

2.2 Value of innovations and structure of information

The social value of an innovation, v, is the total surplus the new product generates. To focus on the factor market, we will assume innovators are perfect price discriminators in the final goods market, which means the revenue of each innovator coincides with the social value of her innovation.

[1] CES production functions exhibit a property called returns to specialization (or love for variety in the case of utility functions). Following an argument similar to Ethier (1982), suppose (1) represents a production function, and let X be the total quantity of inputs used in production. Because of symmetry, all inputs will be used in the same quantity in equilibrium, so $x_i = X/n$ for all i and $y = a(n)X$. If $a'(n) > 0$, there are positive returns to specialization because an increase in n holding X constant causes output to increase. CES production functions of this type are widely used in models of imperfect competition, trade, and growth (see Kim, 2004, for a review).

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Different innovations have different social values. Suppose innovators are indexed by the return to their innovations, which ranges between $\underline{v}$ and $\bar{v}$ with cumulative distribution $F(v)$. Then $F(v)$ is the measure of innovations with a return lower than or equal to v. Also assume the input sellers cannot price discriminate between the innovators.

The hazard function is defined as $h(v) = f(v)/(1-F(v))$, where $f(v)$ is the density function corresponding to $F(v)$. Assumption 1 guarantees the quasi-concavity of the maximization problem of the input producers.

Assumption 1 (Non-decreasing hazard function)**.** *$h(v) > 0$ and $h'(v) \geq 0$ on a support $[\underline{v}, \bar{v}]$, and $h(v) = 0$ outside of this support.*

Assumption 1 is analogous to the one found in Lerner and Tirole (2004), and holds for a great variety of continuous distribution functions. Notice we are not restricting $\underline{v}$ nor $\bar{v}$ to be of finite value.

To understand the meaning of Assumption 1, let c be the cost of R&D for inventing a new good. Then $h(c) = -\frac{\partial(1-F(c))}{\partial c}\frac{1}{(1-F(c))}$ measures the proportional decrease in innovation activity when the cost of R&D increases. In other words, the hazard rate is the *semi-elasticity* of innovation with respect to cost, expressed in absolute value. Assumption 1 only requires the sensibility of innovation to the cost of R&D to increase (or at least not decrease) as cost increases.

An important assumption is that $F(v)$ does not change with n. This assumption, together with the absence of returns to specialization in the R&D technology, leads to the result in Lemma 1. The proofs of all theorems are in the Appendix.

Lemma 1. *The measure of socially desirable innovations does not depend on the level of technological complexity.*

Lemma 1 is important because it allows us to compare the profitability of innovations with the same net social value but different n. In section 3.5 we relax these assumptions by letting the value of innovations be a function of n and allowing returns to specialization in the R&D technology. We find a change in these assumptions does not significantly affect the main results of the paper.

2.3 Interpretation of the innovation technology

The CES specification for the R&D technology is a simple and general way to introduce substitutability and complementarity between the inputs used in research. In our model, ideas have economic value because they are embodied in physical objects (Romer, 1990, Boldrin and Levine, 2002, 2005b). The innovator uses these physical objects –not the abstact ideas– to innovate. Accordingly, the input decision is not discrete (to use the idea or not), but rather is continuous (the research inputs can be used in variable amounts).

Our description of the R&D process is a good description of several innovation technologies. First, we can think innovators use inputs in variable amounts in R&D, and once they perform the innovation, they no longer need those inputs. This interpretation fits well for sectors that use a large number of research tools, such as biomedical research, where R&D requires the use of clones and cloning tools, laboratory equipment and machines, reagents, computer software, and many other research instruments in variable amounts.

The second interpretation is that inputs are actually components of the final product and are used to produce each copy of it. This interpretation more closely resembles the case of already-patented code lines used in new software, hardware components for computers, and a variety of cases in electronics, semiconductors, and other similar industries.[2]

Under this second interpretation, a continuum of perfectly competitive innovators exists. The production function of output is (1), and perfect competition ensures the price of the final output is equal to its marginal cost $c = \sum_{i=1}^{n} p_i x_i$. The demand of the final good is $y = 1 - F(c)$, where y and c are the quantity and price of the final good.

The two interpretations lead to the same results, except that in the second one we see a welfare loss stemming from the non-competitive pricing in the inputs market, which is approximately equal to $(c-\varepsilon)(F(c)-F(\varepsilon))/2$. For expositional purposes, we will stick with the first interpretation.

Alternatively, we could think innovators pay only for the permission to use the idea, in which case assuming input choices are discrete (1 if the input is used and 0 otherwise) would be more appropriate. This approach is the one Lerner and Tirole (2004) follow. In this sense, we show Lerner and Tirole's results can be extended to the continuous case.

[2]In the early radio industry, for example, according to Edwin Armstrong (inventor of FM radio) "it was absolutely impossible to manufacture any kind of workable apparatus without using practically all of the inventions which were then known."

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2.4 Market interaction

The players of the game are the n input sellers and the innovators. A strategy for input seller i is a choice of price for her input. A strategy for each innovator is a function $g : \mathbb{R}^n_+ \times [\underline{v}, \bar{v}] \to \mathbb{R}^n_+$, namely, a demand x_i for each input, as a function of the price of all the inputs and the value of her innovation.

The timing of the game is as follows: (i) input producers simultaneously set the price of their inputs; and (ii) given prices, innovators calculate the input mix that minimizes the cost of innovation, and decide whether to innovate.

The equilibrium concept is symmetric subgame perfect equilibrium (SSPE).[3] A set of strategies $\{p_i\}_{i=1}^n, g$ is a SSPE if it is a Nash equilibrium of every subgame of the original game, and $p_i = p$ for all i. Finally, the payoff for input producer i is $x_i\,(p_i - \varepsilon)$ and the payoff of each innovator is $I\,v - \sum_{i=1}^n p_i\,x_i$.

2.4.1 Innovators' problem

Given input prices $\{p_i\}_{i=1}^n$, each innovator minimizes the cost of innovation, $c = \sum_{i=1}^n p_i\,x_i$, subject to $y \geq 1$. The solution to this problem, common to all innovators, is the set of conditional factor demands x_i and the minimum cost of innovation c. Given c, only innovations whose value is larger than the cost ($v \geq c$) will be performed ($I = 1$).

2.4.2 Input seller's problem

When setting prices, input sellers know that given $\{p_i\}_{i=1}^n$ the measure of performed innovations will be $1 - F(c)$. Therefore, the total demand of input i is $X_i = (1 - F(c))\,x_i$, and the profit maximization problem is

$$\max_{p_i} \quad \Pi_i = (1 - F(c))\,x_i(p_i - \varepsilon),$$

where c and x_i come from the innovators' cost-minimization problem.

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[3] Asymmetric equilibria, in which inputs sellers choose different prices, may also exist. In this type of equilibria, innovators would use a heterogeneous mix of research inputs, which would imply the total amount of resources (i.e., the social cost) needed to perform an innovation is greater than ε. Therefore, by focusing on the symmetric equilibrium, we are focusing on the equilibrium that minimizes the social cost of innovation, and we are guaranteeing the social cost of innovation remains constant as σ or n change.

3 Equilibrium

In this section, we solve recursively for the SSPE. The innovators' problem is a standard cost-minimization problem, from which we get the conditional input demands and the cost function. In equilibrium, all innovators choose $y = 1$ and use all the research inputs in R&D.[4]

Given the conditional demands and the cost function, the problem of input seller i is to choose a price p_i to maximize profits. Let p_i^e denote the equilibrium price of input seller i. In the symmetric equilibrium, $p_i^e = p^e$ for all i, and $p^e \geq \varepsilon$.[5]

In the Appendix, we show that a SSPE with innovation ($p^e < \bar{v}$) exists and is unique (Lemma A1). The equilibrium price solves

$$\frac{p^e - \varepsilon}{p^e} = \left(\frac{n-1}{n}\sigma + \frac{1}{n}p^e h(p^e)\right)^{-1}. \tag{2}$$

The conditional input demand is $x^e = 1/n$, the cost of innovation is $c^e = p^e$, and the measure of innovation is $1 - F(p^e)$.

In the Appendix, we show the cost of innovation is monotonically decreasing in σ (Lemma A2), which is due to the increase in competition as inputs become more substitutable. As $\sigma \to \infty$, price converges to marginal cost ε, which is the standard Bertrand price competition result with homogeneous goods.

3.1 Complements and substitutes

We classify inputs as market complements and substitutes according to the sign of the cross-price derivative of demand, which, in our setting, is equivalent to analyzing the cross-price derivative of profit.

Definition 1 (Market complements and substitutes)**.** *Input j is a market complement (substitute) of input i if $\partial X_i/\partial p_j < 0$ ($\partial X_i/\partial p_j > 0$).*

An increase in the price of input j has two effects on the total demand of input i. On one hand, as the relative price of input i decreases, its conditional demand

[4] Even though innovators could choose $x_i = 0$ for some i when $\sigma > 1$ and still get $y > 0$, they never will in the symmetric equilibrium. When $p_i^e = p^e$ for all i, cost minimization always leads to a symmetric input mix.

[5] Because of the nature of Nash equilibria, for any value of n, ε, and $\sigma < \infty$ equilibria exist for which p^e is so high that nobody innovates (i.e., profits are zero for all input sellers), but any deviation by a *single* input seller is not enough to make innovation profitable. However, we are interested in the existence of equilibria *with* innovation ($p^e < \bar{v}$).

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increases (*substitution effect*). This effect is positive, except in the case of perfect complements, when it is equal to zero. On the other hand, the measure of privately profitable innovations $(1-F(c^e))$ decreases because the inputs are more expensive and fewer innovators will find performing the innovation profitable (*innovation effect*). This effect is negative, except in the case of perfect substitutes, when it is equal to zero. The sign of the cross-price derivative depends on which of the two effects is stronger.

In what follows, when we say inputs are complements or substitutes, we mean they are *market* complements or substitutes. We will see the distinction between complements and substitutes is crucial for the predictions of the model.

Lemma 2. *The cross-price derivative $\partial X_i/\partial p_j$ is zero in the symmetric equilibrium if and only if $\sigma = \sigma^*$, where σ^* is the argument that solves $h(\sigma^* \varepsilon/(\sigma^*-1)) = \sigma^*-1$. In the symmetric equilibrium, inputs are complements when $\sigma < \sigma^*$ and substitutes when $\sigma > \sigma^*$.*

Notice that since the *innovation effect* works in favor of complementarities, the value of σ which divides inputs into complements and substitutes has to be larger than or equal to 1. As a proof, suppose $\sigma^* < 1$. Then $h(\varepsilon\sigma^*/(\sigma^*-1)) < 0$, which is not possible. Notice also that σ^* does not depend on n. This result changes in section 3.5, where we allow the social cost and the social value of the innovation to vary with n.

3.2 Increasing complexity

Proposition 1 shows the sign of the effect of an increase in the complexity of the innovation, n, depends on whether the inputs are complements or substitutes.

Proposition 1. *The cost of innovation increases as innovation becomes more complex if the inputs are complements ($\sigma < \sigma^*$) and decreases as innovation becomes more complex if the inputs are substitutes ($\sigma > \sigma^*$).*

The measure of performed innovations is $1-F(c^e)$, so innovation activity and cost move in opposite directions: if inputs are complements (substitutes), innovation activity decreases (increases) as n increases. Proposition 1 is one of the central results of the paper, which we discuss in greater detail in section 3.4.

3.3 High complexity

In this section, we analyze what happens as $n \to \infty$. Let p_∞ be the equilibrium price of the limiting economy. Proposition 2 characterizes equilibria with innovation ($p_\infty < \bar{v}$). In this case, for some values of σ, no equilibrium with innovation exists.[6]

Proposition 2. *When $n \to \infty$, a SSPE with innovation exists only for $\sigma > \hat{\sigma} = \frac{\bar{v}}{\bar{v}-\varepsilon}$, where $1 \leq \hat{\sigma} < \sigma^*$. The equilibrium price and cost of innovation are $p_\infty = \frac{\sigma}{\sigma-1}\varepsilon$.*

The equilibrium of the limiting economy does not depend on the distribution of v, but it depends on the upper bound of the support of the distribution. When inputs are substitutes, the equilibrium price is the same as in Dixit and Stiglitz's 1977 monopolistic competition model: firms set a mark-up over marginal cost equal to $\sigma/(\sigma-1)$. Therefore, the pricing inefficiency decreases as n increases, but it does not disappear even when $n \to \infty$.

For complements, the outcome depends on whether σ is greater or smaller than $\hat{\sigma} = \bar{v}/(\bar{v}-\varepsilon)$. When $\sigma > \bar{v}/(\bar{v}-\varepsilon)$, firms set a mark-up just like in the substitutes case. When $\sigma \leq \bar{v}/(\bar{v}-\varepsilon)$, the only equilibria have $p_\infty \geq \bar{v}$, so innovation activity is null. In this case, as n increases, the inefficiency due to monopoly pricing increases and is at its maximum when $n \to \infty$.

3.4 The tragedy of the anticommons revisited

The model we presented in the above sections gives a formal treatment to the tragedy of the anticommons in sequential innovation. Anticommons arise when multiple owners have the right to *exclude* each other from using a scarce resource, causing its inefficient under-utilization (Heller, 1998). This problem is the dual of the tragedy of the commons, in which multiple owners have the right to *use* a scarce resource, and resources tend to be overused (Buchanan and Yoon, 2000).

Heller and Eisenberg (1998) claim the tragedy of the anticommons may be severely acute in biomedical research, but other hi-tech industries where innovation follows a similar cumulative process may share the same problem. Specifically, Heller and Eisenberg point out that the excessive fragmentation of intellectual property rights on research tools may reduce the incentive to innovate, because the innovator has to face a possibly high cost of bundling all the licenses together. This high cost may be the consequence of transaction costs as well as strategic behavior over the distribution of the surplus.

[6] Any $p_\infty \geq \bar{v}$ is an equilibrium for any value of σ in this limiting economy. If $p_\infty \geq \bar{v}$, no innovation occurs. If a single input seller deviates, her impact on the cost of innovation is infinitesimal, and does not affect innovation. Therefore, no profitable deviations exist when $p_\infty \geq \bar{v}$.

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Proposition 1 shows that, when inputs are market complements, the cost of producing an innovation increases as technologies become more complex. Therefore, the anticommons hypothesis applies to sequential innovation when research inputs are market complements. The opposite is true when research tools are market substitutes, in which case we see no anticommons because, as the number of research inputs increases, competitive pressure reduces their price fast enough to also reduce the cost of innovation.

Proposition 2 reinforces the previous results. When the number of patented research tools is very large and inputs are highly complementary, the anticommons is so strong that innovation activity stops completely.

Finally, note that in our model the anticommons effect arises in the absence of any kind of transaction costs, as a natural consequence of the uncoordinated market power of the input producers. Introducing transaction costs would simply reinforce the anticommons problem stemming from uncoordinated pricing.

3.5 Social value or social cost depend on complexity

So far, we have assumed the distribution of values of the innovation and the social cost of innovation do not depend on n. Under these assumptions, a change in n only changes the number of producers from whom innovators have to buy research inputs, but does not change the measure of socially valuable innovations. In this section, we study what happens if we assume the social value or the social cost of innovation varies with n.

Remember $a(n)$ in equation (1) measures the returns to specialization. Let $a'(n) \neq 0$. If $a'(n) > 0$, specialization produces positive returns and the social cost of innovation decreases as n increases. If $a'(n) < 0$, specialization produces negative returns and the social cost of innovation increases with n (see footnote 1).

Also let the value of innovation be $b(n)\,v$, where v has a cumulative distribution $F(v)$ as in the previous sections, and $b(n) \geq 0$ is a scale factor determining the effect of complexity on the value of the innovations. If $b'(n) > 0$ then innovations become more valuable as complexity increases. If $b'(n) > 0$ innovations become less valuable as complexity increases.

It is straightforward to show the measure of performed innovations is now $1 - F(\tilde{c})$, where $\tilde{c} = c/(a(n)\,b(n))$. Let $\tilde{\varepsilon} = \varepsilon/(a(n)\,b(n))$ and $\tilde{\varepsilon}_\infty = \lim_{n\to\infty}\tilde{\varepsilon}$. We are interested in determining the effects of changes in n on $\tilde{c}$, since this measure determines the probability of innovation. Notice we are not imposing an upper bound on $\tilde{\varepsilon}$, but $\tilde{\varepsilon}_\infty \geq 0$ since $a(n)\,b(n) \geq 0$. Equilibrium cost solves

$$\frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e} = \left(\frac{n-1}{n}\sigma + \frac{1}{n}h(\tilde{c}^e)\,\tilde{c}^e\right)^{-1}. \qquad (3)$$

This equilibrium is equivalent to the one in the basic model, thinking of $\tilde{c}^e$ as the cost of innovation and $\tilde{\varepsilon}$ as the social cost of the inputs. In other words, the equilibrium is equivalent to the one we would obtain if we assumed ε changes with n in the original model.

Given that equation (3) is mathematically equivalent to equation (2), the distinction between complements and substitutes is still determined by σ^* as given in Lemma 2. The only difference is that now σ^* may increase or decrease with n, depending on the sign of $\tilde{\varepsilon}'$.

The following proposition shows that when social cost or social value depend on n, the threshold value of σ determining the effects of complexity on innovation activity differs from σ^*. Specifically, results depend on $\gamma(n) = -n\,\tilde{\varepsilon}'(n)/\tilde{\varepsilon}(n)$, which measures the concavity of $\tilde{\varepsilon}$. As an example, consider $a(n) = n^\theta$ and $b(n) = n^\psi$, which is the usual parameterization used with CES production functions. In this case, $\gamma(n) = \theta + \psi$. For easiness of exposition, in what follows we refer to $\gamma(n)$ as γ, but keep in mind that γ may depend on n.

Proposition 3. *If $\gamma > 0$, $\tilde{\sigma} \leq \sigma^*$ exists such that the cost of innovation increases as innovation becomes more complex for $\sigma < \tilde{\sigma}$ and decreases as innovation becomes more complex for $\sigma > \tilde{\sigma}$. $\tilde{\sigma} > 0$ if and only if $h((1+\gamma)\tilde{\varepsilon}) < n\,(\gamma\tilde{\varepsilon})^{-1}$. If $\gamma < 0$, and γ is small in absolute value, $\tilde{\sigma} \geq \sigma^*$ exists such that the cost of innovation increases as innovation becomes more complex for $\sigma < \tilde{\sigma}$ and decreases as innovation becomes more complex for $\sigma > \tilde{\sigma}$.*

Comparing Propositions 1 and 3, we can see that allowing the social cost or social value of innovation to change with n has an effect on the threshold value of σ determining whether increases in n encourage or discourage innovation. If $\gamma = 0$, this threshold value coincides with the one that separates research inputs into complements and substitutes. If $\gamma > 0$, the measure of socially optimal innovations increases with n, so the threshold value shifts to the left. If $\gamma < 0$ (and γ is small in absolute value), the measure of socially optimal innovations decreases with n, so the threshold value shifts to the right.[7]

To conclude with this section, we analyze what happens when $n \to \infty$. Let $\tilde{c}_\infty = \lim_{n\to\infty} \tilde{c}^e$. As in section 3.3, a SSPE with innovation ($\tilde{c}_\infty < \bar{v}$) exists only if $\sigma > \hat{\sigma}$, where $\hat{\sigma} = \frac{\bar{v}}{\bar{v}-\tilde{\varepsilon}_\infty}$, in which case the equilibrium cost of innovation would be $\tilde{c}_\infty = \frac{\sigma}{\sigma-1}\,\tilde{\varepsilon}_\infty$. Interestingly, if $\tilde{\varepsilon}_\infty = 0$, then $\tilde{c}_\infty = \bar{v}$ for $\sigma \leq 1$ and $\tilde{c}_\infty = 0$ for $\sigma \geq 1$.

[7] If γ is negative and large in absolute value, the direct effect of n on $\tilde{\varepsilon}$ dominates all other effects, and innovation activity decreases with n.

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4 Patent policy

We have shown the fragmentation of intellectual property rights may harm innovation activity when research inputs are complementary, which is the result of uncoordinated market power of the input sellers. The policy maker could reduce the anticommons by eliminating the coordination problem (we will analyze this option in section 5) or by reducing market power (reducing patent breadth or length). However, weaker patents also imply a lower incentive to discover research inputs. The optimal patent policy should balance these two effects. In this section, we study the optimal response of patent policy to increasing complexity.

The innovation technology is the same as in section 2. For tractability, we will focus on the perfect complements case; that is, the new final goods can only be invented if all research inputs have already been invented. Without loss of generality, we assume that once the inputs have been invented, they can be reproduced at zero marginal cost ($\varepsilon = 0$). Finally, we focus on the uniform distribution case, $v \sim U[0,1]$, which implies a linear demand for the research inputs.

An important difference with section 2 is that now the n research inputs must be invented at an earlier stage. The fixed cost of inventing an input is K/n. An input will be introduced if expected revenues are larger than the fixed cost. As is standard in the literature of sequential innovation, the fixed cost of the input sellers is unknown to the policy maker. All the policy maker knows is that the sunk cost has a distribution $K \sim U[0,\bar{K}]$. Therefore, the patent policy cannot depend on the realization of K. Our assumptions imply the social cost and the social value of the innovations do not change as n increases. All that changes is the number of research inputs used in R&D.

Let $\phi \in [0,1]$ be a policy parameter measuring the probability that an innovator is granted a patent and that the patent can be successfully defended in court. Imitation is costless, and Bertrand competition ensures that innovations not protected by patents are sold at marginal cost. The patent policy applies equally to the input innovators and to the final good innovators. The expected revenue of a final innovator is ϕv, where $v \sim U[0,1]$ is the gross social value of the innovation. Innovators only have to pay a non-competitive license fee for patented inputs, and can buy non-patented inputs at marginal cost.

The timing of the game is as follows: (i) input innovators observe the cost of innovation and decide whether to invent their input; (ii) if all inputs are invented, Nature decides which inputs are protected by patents; (iii) patent holders set a price for patented inputs; (iv) final innovators decide whether to innovate; and (v) Nature decides which final innovations are protected by patents.

A research input will be invented only if the expected revenue from selling the input is higher than the cost of inventing it. Actual revenue will depend on whether the input is covered by a patent and on how many other inputs are covered by patents. Suppose m patents are granted in the second stage. Then the price of the inputs, the cost of innovation, and the conditional measure of final innovations performed (conditional on the n inputs being invented) are $p_m^e = \frac{\phi n}{m+1}$, $c_m^e = \frac{\phi m}{m+1}$, and $1 - F(c_m^e/\phi) = \frac{1}{m+1}$. Interestingly, input producers completely internalize the effect of ϕ on the profits of final innovators, so the conditional measure of innovation activity does not depend on patent strength.

To find the unconditional measure of innovation activity, we need to solve the first stage of the game. Consider an input innovator who is granted a patent. In the third stage, her revenues depend on how many other patents have been granted. Let $\ell = m-1$ denote the number of patents granted, in addition to the patent of the input innovator we are considering. Actual revenues (after uncertainty is resolved) are $\Pi_\ell = \phi/(\ell+2)^2$. Expected revenues are

$$E(\Pi) = \phi \sum_{\ell=0}^{n-1} \frac{\phi}{(\ell+2)^2} \frac{(n-1)!}{(n-1-\ell)!\ell!} \phi^\ell (1-\phi)^{n-1-\ell}. \qquad (4)$$

An increase in ϕ (i) increases the probability of being granted a patent, (ii) raises input price, and (iii) increases the probability that more patents are granted to other innovators. While (i) and (ii) increase $E(\Pi)$, (iii) decreases $E(\Pi)$ because of the anticommons effect.

Let us now focus on how patent policy affects expected innovation. A final innovation will be introduced if (i) expected revenues are larger than the fixed cost for all input sellers, and (ii) the value of the innovation for the final innovator is larger than the cost of paying the inputs protected by patents. The probability that (i) happens is $Pr(E(\Pi) > K/n) = nE(\Pi)/\bar{K}$. Given m, the measure of final innovations introduced is $1 - F(c_m^e/\phi) = \frac{1}{m+1}$. Therefore, the expected measure of final innovations is

$$E(1 - F(c_m^e/\phi)) = \sum_{m=0}^{n} \frac{1}{m+1} \frac{n!}{(n-m)!m!} \phi^m (1-\phi)^{n-m}.$$

Expected innovation activity is the expected measure of final innovations performed weighted by the probability that all inputs are invented. Remember that innovation in a second-generation product is only possible if all inputs have already been invented. Expected innovation is

$$EI = \frac{nE(\Pi)}{\overline{K}} \left(\sum_{m=0}^{n} \frac{1}{m+1} \frac{n!}{(n-m)!m!} \phi^m (1-\phi)^{n-m} \right). \qquad (5)$$

Figure 1 shows expected innovation increases with ϕ, reaches a maximum (at the optimal policy ϕ^*), and then decreases. Two effects are pulling in opposite

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directions: on one hand, increasing ϕ increases expected revenues of final innovations, but on the other hand, it also increases their cost. Proposition 4 determines the optimal policy as a function of n.

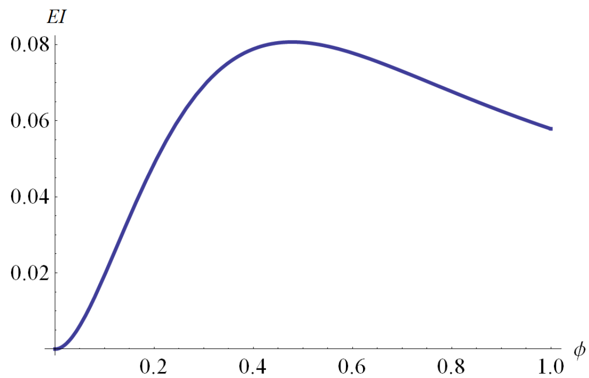

Figure 1: Probability of final innovation. n=5, K=0.4

Proposition 4. *The optimal patent policy parameter (ϕ^*) is decreasing in the complexity of innovation.*

In models of sequential innovation, the degree of patent protection determines the division of profits between sequential innovators. Stronger patents increase the protection for early innovators, and allows them to claim a larger part of the surplus of subsequent innovators. However, when innovations build on several prior complementary inventions, the uncoordinated market power of earlier patentees generates an anticommons effect. Proposition 4 shows that, as the number of claims increase, the optimal response is to reduce the degree of patent protection. Note that in our model, a minimum amount of protection is always needed; otherwise, nobody would have an incentive to innovate.

4.1 Patent policy with imperfect substitutability

In this section, we allow the substitutability between research inputs to be higher than zero. Any degree of substitutability higher than one requires a positive marginal cost ε for the inputs; otherwise, innovators would only use unpatented inputs and would be able to innovate at zero cost when inputs are substitutes. As before, inputs require a fixed cost of K/n to be invented, with $K \sim U[0,\bar{K}]$ and $v \sim U[0,1]$.

This new setting generalizes the analysis of section 4. However, the problem becomes analytically intractable and we have to resort to numerical simulations.

The results of the numerical simulations are in line with those of section 4. The optimal patent strength is decreasing in complexity when inputs are complements. Figure 2 shows the optimal patent strength ϕ^* as a function of the elasticity of substitution σ, with $\varepsilon = 0.1$ and $\bar{K} = 0.4$, for $n = 5$ and $n = 10$. Increasing n reduces ϕ^* for low σ, and increases ϕ^* for high σ.

Our numerical simulations also show that ϕ^* is non-decreasing in σ. A higher elasticity of substitution increases competition among input producers, reducing their ability to set a price above the marginal cost. Therefore, an increase in σ redistributes revenues from input producers to final innovators. To compensate for this effect, patents must be strengthened. Notice, however, that when σ is large, patents provide limited market power anyway because competition between input sellers is very intense. Finally, note some value of σ larger than 1 exists for which the optimal patent policy is 1; that is, inputs should be protected by very strong patents when the substitutability is large enough.

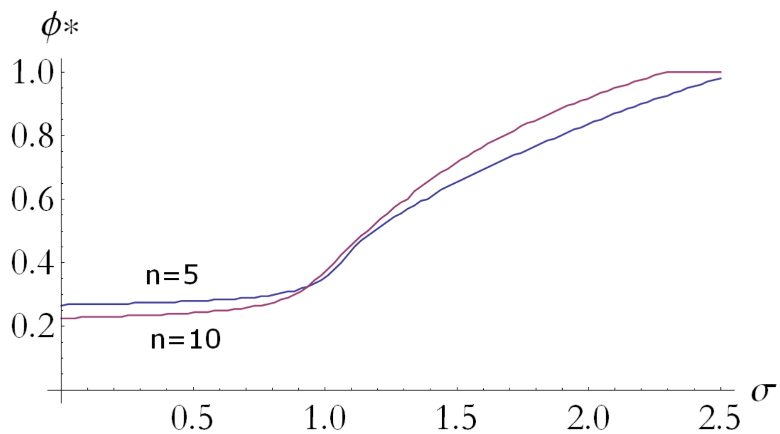

Figure 2: Complexity and optimal patent policy for imperfect substitution

5 Patent pools

Even though our objective is to study the optimal patent policy, analyzing what would happen if research inputs were priced cooperatively, either by a collective institution such as a patent pool or by a single patent holder (monopolist) that owns all the patents, would also be of interest.

For this analysis, we come back to the basic model of section 2 (innovators require n inputs, which have already been invented and protected by patents).

Proposition 5 shows the equilibrium price in this case and compares it with the equilibrium price in the non-collusive equilibrium. The difference is that now the patent holder maximizes joint-profits and therefore takes into account the cross-price effects between input demands.

In what follows, p^* denotes the equilibrium price when inputs are priced cooperatively. As in previous sections, p^e denotes the non-cooperative price defined in equation (2).

Proposition 5 (Patent Pool). *The cooperative price solves* $h(p^*)(p^* - \varepsilon) = 1$. $p^* = p$ *if and only if* $\sigma = \sigma^*$, $p^* < p^e$ *if and only if inputs are complements (*$\sigma < \sigma^*$*), and* $p^* > p^e$ *if and only if inputs are substitutes (*$\sigma > \sigma^*$*).*

Notice that p^* depends only on the functional form of h and on the value of ε, but not on the values of σ or n.

Our results on patent pools are similar to those of Lerner and Tirole (2004). The difference between the two papers is that we have assumed inputs are used in research in a continuous fashion, whereas Lerner and Tirole (2004) consider discrete input choices (1 if the input is used and 0 otherwise).

Under the latter approach, the equilibrium will depend on whether the competition margin or demand margin bind. When the competition margin binds, if the input seller raises her price, her input would be evicted from the bundle of patents the final innovator buys. When the demand margin binds, the input seller can raise price without excluding its input from the basket, but the overall demand for the bundle would decrease.

The competition and demand margins are related to our substitution and innovation effects. The substitution effect says an increase in the price of one of the inputs lowers the demand for that input and increases the demand for other inputs (holding overall demand constant). The innovation effect says an increase in the price of an input lowers the overall demand for the basket of inputs (holding the relative demand of the inputs constant).

Therefore, our model can be interpreted as a "smooth" version of Lerner and Tirole's. In Lerner and Tirole, as inputs become more complementary, the demand margin is more likely to bind. In our model, both margins always bind (except when $\sigma = 0$ or $\sigma \to \infty$), but as inputs become more complementary, the innovation effect becomes more important relative to the substitution effect. On the other hand, as σ increases, the substitution effect becomes more important, limiting and eventually eliminating the anticommons.

6 Extensions

In this section we analyze the consequences of relaxing some of the basic assumptions of the model.

6.1 No price discrimination

We can relax the assumption that innovators are perfect price discriminators. Dropping this assumption introduces a wedge between the social and private values of each innovation, which means the distribution of values of innovation changes and that the final goods sector also contains an inefficiency. Assume the social value of the innovations is still distributed according to $F(v)$, with probability density function $f(v)$. The private value of each innovation is now v_p, which is less than its social value. With a linear demand for the final good, for example, the private return of innovations would be $v_p = v/2$, and the probability density function would be $2f(2v_p)$. The qualitative results are the same as before. All that changes is that now innovation activity decreases for each value of σ, and so the values of σ^*, $\hat{\sigma}$, and $\bar{\sigma}$ increase. Also, the optimal patent protection is lower for each value of σ and n than in the case of perfect price discrimination.

6.2 Patent breadth and degree of substitutability

We have assumed the degree of substitutability between the inputs is exogenous. However, σ could be related to a patent policy instrument, such as patent breadth. A narrower patent breadth allows imitators to enter the market and produce similar products, limiting the market power of the input innovator. In the basic model, having very narrow patents would then be optimal, so as to increase competition and drive the price of the inputs to the marginal cost. But then one should take into account that narrower patents would reduce the incentive to invent the inputs, and would also affect final good innovators.

Even though we do not explicitly take this possibility into account, such extension would generate a similar trade-off to the one analyzed in section 4, and thus would lead to similar results.

6.3 Alternative interpretation of the innovation process

Instead of having a continuum of innovators, one can think only one potential innovator is present. v is the value of the innovation, which is private information of the innovator. Input sellers know v has a cumulative distribution function equal

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to $F(v)$. Under this interpretation, we should add an intermediate stage in the timing of the game: Nature draws the value of innovation after input sellers set the price and before the innovator decides whether to innovate. It is easy to see that all the previous results translate directly into this setting. All that changes is that now $1-F(c)$ is not the measure of innovations but the ex-ante probability that the innovation is performed.

7 Conclusion

In this paper, we extend work by Shapiro (2001) and Lerner and Tirole (2004) in three directions: (i) we study what happens with innovation activity as the number of patented inputs potentially involved in the innovation process increases; (ii) we determine the optimal patent policy as a function of the number of research inputs; and (iii) we consider not only the incentives to create the new final good, but also the incentives to create the research inputs (ex-ante effect). By doing so, we connect the literatures of complementary monopoly and patent pools to the literature of sequential innovation.

Two key insights of the paper are: (i) increasing complexity may have a negative effect on innovation activity when research inputs are complementary, and (ii) either limiting market power through weaker patents or reducing the lack of coordination through the creation of patent pools may solve this problem. In particular, the optimal patent breadth is increasing in the elasticity of substitution between the inputs used in research, and decreasing (increasing) in the complexity of the R&D process when research inputs are complements (substitutes).

Our findings contribute to the recent debate on the emergence of anticommons in processes of sequential innovation (Heller, 1998, Heller and Eisenberg, 1998). Concretely, our model identifies the conditions under which anticommons may arise (complementarity and increasing complexity), and proposes policy solutions to such episodes. We hope this paper can stimulate further research on this important issue, both at a theoretical and at an empirical level.

Appendix: Proofs of propositions and lemmas in text

Proof of Lemma 1. An innovation is socially desirable if its social value is larger than or equal to its social cost. The social cost of an innovation coincides with the resources used to produce it. Therefore, the measure of socially desirable innovations is $1-F\left(\sum_{i=1}^{n}\varepsilon x_i\right)$. Because of the symmetry in the innovation technology, the socially optimal choice of inputs is $x_i = 1/n$, so this measure becomes

$1-F(\varepsilon)$, which depends on the distribution of social values of the innovations and the marginal cost of the inputs but *not* on the number of inputs used in R&D. ■

Proof of Lemma A1 (existence and uniqueness of SSPE) . The conditional demand of input i and the cost of innovation for each innovator are

$$x_i = I\, n^{-\frac{1}{1-\sigma}}\, p_i^{-\sigma} \left(\sum_{i=1}^{n} p_i^{\,1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \qquad c = n^{-\frac{1}{1-\sigma}} \left(\sum_{i=1}^{n} p_i^{\,1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

Input producer i wants to maximize $\Pi_i = (1-F(c))\, x_i\, (p_i - \varepsilon)$. The derivative with respect to price is

$$\Pi_i'(p_i) = -f(c)\,\frac{\partial c}{\partial p_i} x_i\,(p_i-\varepsilon) + (1-F(c))\left(\frac{\partial x_i}{\partial p_i}\,(p_i-\varepsilon) + x_i\right).$$

By Shepard's Lemma $\partial c/\partial p_i = x_i$, and by symmetry $c = p$, $x_i = 1/n$, and $\partial x_i/\partial p_i = -(n-1)\sigma/(n^2 p)$. Therefore, the first order condition becomes

$$\Pi_i'(p) = -f(p)\,\frac{p-\varepsilon}{n^2} + (1-F(p))\left(-\frac{\sigma\,(n-1)\,(p-\varepsilon)}{n^2\,p} + \frac{1}{n}\right).$$

Now we prove that the solution cannot be ε nor $\bar{v}$ for $n < \infty$. $p = \varepsilon$ cannot be the equilibrium because $\Pi_i'(\varepsilon) = (1-F(\varepsilon))/n > 0$. Also, $p = \bar{v}$ cannot be the equilibrium both if $\bar{v}$ is finite or infinite. If $\bar{v} < \infty$, then $\Pi_i'(\bar{v}) = -f(\bar{v})\left(\frac{\bar{v}-\varepsilon}{n}\right) < 0$. On the other hand, $lim_{p\to\infty}\Pi_i'(p) = -\infty < 0$. Therefore, the solution must satisfy $\Pi_i'(p) = 0$. Multiplying $\Pi_i'(p)$ by $-n^2/(1-F(p))$ we obtain

$$h(p)\,(p-\varepsilon) + \sigma\,(n-1)\,\frac{p-\varepsilon}{p} - n = 0. \tag{6}$$

We can be sure equation (6) has exactly one solution because it is continuously increasing in p by Assumption 1, is negative when $p = \varepsilon$ and is positive when $p \to \bar{v}$ (Assumption 1 implies that $\lim_{p\to\bar{v}} h(p)\,p = \infty$ for finite or infinite $\bar{v}$). Therefore, the solution exists and is unique. Rearranging terms in equation (6) we get equation (2). ■

Proof of Lemma A2 (cost of innovation is decreasing in σ) . Equation (2) provides an implicit function of p^e in terms of σ. We can calculate $\partial c^e/\partial\sigma$ using the implicit function theorem (remember that $p^e = c^e$ in the symmetric equilibrium):

$$\frac{\partial c^e}{\partial \sigma} = -\frac{(n-1)(p^e-\varepsilon)/p^e}{h(p^e) + h'(p^e)\,(p^e-\varepsilon) + \sigma\,(n-1)\varepsilon/p^{e2}}$$

It is easy to see that this derivative is always negative (the numerator and the denominator are positive). The result follows. ■

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Proof of Lemma 2. The cross-price derivative is

$$\frac{\partial E(x_i)}{\partial p_j} = (1 - F(c))\,\frac{\partial x_i}{\partial p_j} - f(c)\,x_i.$$

By Shepard's Lemma, $\partial c/\partial p_j = x_j$. Imposing symmetry, $x_i = x_j = 1/n$, and $\partial x_i/\partial p_j = \sigma/(n^2\,p)$. Rearranging terms we obtain

$$\frac{\partial E(x_i)}{\partial p_j} = \frac{1}{n\,(1 - F(p))}\left(\frac{\sigma}{p} - h(p)\right).$$

This derivative will be equal to zero in equilibrium only when $h(p^e) = \sigma/p^e$. Introducing this expression into the first order condition (2) and rearranging we obtain $(p^e - \varepsilon)/p^e = \sigma^{-1}$, or $p^e = \sigma\varepsilon/(\sigma - 1)$. Plugging this value of p^e into $h(p^e) = \sigma/p^e$ we obtain the expression $h\left(\frac{\sigma}{\sigma-1}\varepsilon\right) = \sigma - 1$.

Now we know the cross-price derivative is zero when $\sigma = \sigma^*$ and that its sign depends on $\sigma/p^e - h(p^e)$. The latter expression is increasing in σ because p^e is decreasing in σ, and h is non-decreasing in p^e from Assumption 1. Hence inputs are complements when $\sigma < \sigma^*$ and substitutes when $\sigma > \sigma^*$. ■

Proof of Proposition 1. We are looking for the effect of a unit increase in n, but it will suffice to determine the sign of $\partial c^e/\partial n$. Equation (2) provides an implicit function of c^e in terms of n. Therefore, we can calculate $\partial c^e/\partial n$ using the implicit function theorem:

$$\frac{\partial c^e}{\partial n} = \frac{1 - \sigma\,(p^e - \varepsilon)/p^e}{h'(p^e)\,(p^e - \varepsilon) + h(p^e) + \sigma(n-1)\varepsilon/p^{e2}}$$

We know the denominator is always positive. Therefore, the sign of this derivative depends on the sign of the numerator.

From equation (2) we get the following relation in equilibrium $\sigma\,(p^e - \varepsilon)/p^e = (n - h(p^e)\,(p^e - \varepsilon))/(n-1)$. Introducing this expression in the numerator and operating, the numerator becomes $(h(p^e)\,(p^e - \varepsilon) - 1)/(n-1)$. We know that $h(p^e)\,(p^e - \varepsilon) = 1$ when $\sigma = \sigma^*$. Given that $h(p^e)\,(p^e - \varepsilon)$ is increasing in p^e, it is decreasing in σ. Therefore, the numerator is positive when $\sigma < \sigma^*$ and is negative when $\sigma > \sigma^*$. The result follows. ■

Proof of Proposition 2. From equation (2), we can see that as $n \to \infty$, the term with the hazard function goes to zero. Therefore, the equilibrium price of the limiting economy is $p^e_\infty = \frac{\sigma}{\sigma-1}\,\varepsilon$, which is between ε and $\bar{v}$ only when $\sigma > \bar{v}/(\bar{v} - \varepsilon)$.

It is easy to show that $1 \leq \hat{\sigma} < \sigma^*$. The first inequality follows trivially from the fact that $\hat{\sigma} = \bar{v}/(\bar{v} - \varepsilon)$. Therefore, $\hat{\sigma} = 1$ only when $\bar{v} \to \infty$ or $\varepsilon = 0$. For the second inequality, it is enough to compare the equilibrium price when $\sigma = \hat{\sigma}$ with the equilibrium price when $\sigma = \sigma^*$, since price is decreasing in σ. When $\sigma = \hat{\sigma}$, price is equal to $\bar{v}$. When $\sigma = \sigma^*$ we know that the equilibrium price solves $h(p^e)(p^e - \varepsilon) = 1$. If $p^e = \bar{v}$, then $h(p^e)(p^e - \varepsilon) \to \infty$, which is larger than 1. For $h(p^e)(p^e - \varepsilon)$ to decrease and approach 1, p^e has to decrease. This means that equilibrium price is larger with $\hat{\sigma}$ and therefore $\hat{\sigma} < \sigma^*$. ■

Proof of Proposition 3. We need to find the sign of $\partial \tilde{c}^e / \partial n$. Equation (3) can be rearranged as follows:

$$\frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e} \left(\sigma (n-1) + h(\tilde{c}^e)\, \tilde{c}^e\right) - n = 0. \tag{7}$$

By the implicit function theorem,

$$\frac{\partial \tilde{c}^e}{\partial n} = -\frac{\frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e}\,\sigma - \frac{\tilde{\varepsilon}'}{\tilde{c}^e}\left(\sigma(n-1) + h(\tilde{c}^e)\,\tilde{c}^e\right) - 1}{\left(\sigma(n-1) + h(\tilde{c}^e)\,\tilde{c}^e\right) + \frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e}\left(h(\tilde{c}^e) + \tilde{c}^e\, h'(\tilde{c}^e)\right)\tilde{\varepsilon}/\tilde{c}^{e\,2}}.$$

The denominator is always positive, which means that the sign of $\partial \tilde{c}^e / \partial n$ depends on the sign of the following expression:

$$\lambda = 1 - \frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e}\,\sigma + \frac{\tilde{\varepsilon}'}{\tilde{c}^e}\left(\sigma(n-1) + h(\tilde{c}^e)\,\tilde{c}^e\right)$$

From (3) we obtain $\sigma(n-1) - h(\tilde{c}^e)\tilde{c}^e = n\tilde{c}^e / (\tilde{c}^e - \tilde{\varepsilon})$. Remember $\gamma = -n\tilde{\varepsilon}' / \tilde{\varepsilon}$. Introducing these expressions in λ, we obtain

$$\lambda = 1 - \sigma\,\frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e} - \frac{\tilde{\varepsilon}}{\tilde{c}^e - \tilde{\varepsilon}}\,\gamma. \tag{8}$$

Let us first analyze the case of $\gamma > 0$. Let $\tilde{\sigma}$ be the value of σ for which $\lambda = 0$. We now show that in this case, λ is decreasing in σ, which implies that there is at most one value of σ for which $\lambda = 0$, and that $\tilde{c}^e$ will increase with n for $\sigma < \tilde{\sigma}$ and decrease with n for $\sigma > \tilde{\sigma}$.

From equilibrium condition (3) we get $\sigma(\tilde{c}^e - \tilde{\varepsilon})/\tilde{c}^e = (n - \tilde{c}^e\, h(\tilde{c}^e)\,(\tilde{c}^e - \tilde{\varepsilon})/\tilde{c}^e)/(n-1)$. Introducing in λ and operating, we obtain

$$\lambda = \frac{\tilde{c}^e\, h(\tilde{c}^e)\,\frac{\tilde{c}^e - \tilde{\varepsilon}}{\tilde{c}^e} - 1}{n-1} - \frac{\tilde{\varepsilon}}{\tilde{c}^e - \tilde{\varepsilon}}\,\gamma,$$

which is decreasing in σ because h is increasing in $c\tilde{c}^e$, $\tilde{c}^e$ is decreasing in σ, and $\gamma > 0$.

We now show that when $\gamma > 0$, $\tilde{\sigma} \leq \sigma^*$. We know from the proof of Lemma 2 that $(\tilde{c}^e - \tilde{\varepsilon})/\tilde{c}^e = \sigma^{-1}$ when $\sigma = \sigma^*$. Introducing this expression into λ would eliminate the first two terms, and therefore the sign of λ would be negative. Given that λ is increasing in σ, this means that $\tilde{\sigma} \leq \sigma^*$.

To conclude with the analysis, we would like to know in which cases $\tilde{\sigma} > 0$, because if $\tilde{\sigma} \leq 0$, the cost of innovation decreases with n for any σ (this result is due to the fact that, in this case, the direct effect of the lower social cost of innovation dominates all other effects).

If $\tilde{\sigma} > 0$, then $\lambda > 0$ when $\sigma = 0$. From (8), we know this condition will hold only if $\tilde{c}^e > (1+\gamma)\tilde{\varepsilon}$. On the other hand, from (3) we know that $(\tilde{c}^e - \tilde{\varepsilon})h(\tilde{c}^e) = n$. Combining the two expressions, we have that $\lambda > 0$ when $\sigma = 0$ (i.e. $\tilde{\sigma} > 0$) if and only if $\gamma\tilde{\varepsilon}\, h((1+\gamma)\tilde{\varepsilon}) < n$.

Let us now turn our attention to the case of $\gamma < 0$. This case is slightly more complicated because the derivative of λ with respect to σ may become positive for some values of σ, if γ is large enough in absolute value. The economic reason is that when γ is negative and large in absolute value, the direct effect of the increase in the social cost of innovation dominates all other effects and the private cost of innovation ($\tilde{c}^e$) increases with n, regardless of σ. However, if γ is negative but small, λ will still be decreasing in σ, in which case it is straightforward to show that $\tilde{\sigma} \geq \sigma^*$. ■

Proof of Proposition 4. Maximizing (5) with respect to ϕ we get a first order condition that only depends on n and ϕ:

$$F(\phi,n) = 2a^b(2+n\phi)+f-2-a^{2b}(2+c)+a^n b\phi\left(1+n\phi-a^b(1+c)\right)g(n,\phi)=0,$$

where $a = 1-\phi$, $b = n+1$, $c = \phi+2n\phi$, and $g(n,\phi)$ is the generalized hypergeometric function ${}_3F_2\left(1,1,-n;2,2;\frac{\phi}{\phi-1}\right)$. $F(\phi,n) = 0$ is an implicit function determining the optimal patent policy as a function of n. Figure 3 shows that ϕ^* is decreasing in n. ■

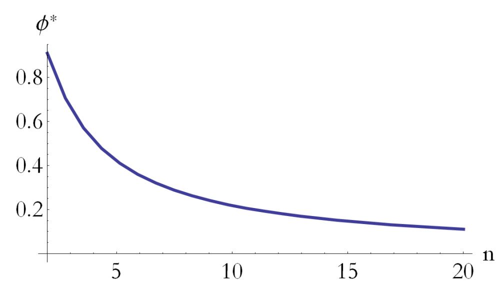

Figure 3: Increasing complexity and optimal patent strength. $K = 0.4$

Proof of Proposition 5. Given the symmetric input demands, the pool wants to sell a symmetric bundle. Therefore $p_i = p$ and $x_i = 1/n$ for all i, and the pool maximizes total profits given by $n\,(1-F(p))\,(p-\varepsilon)$. The first order condition is $n(-f(p)\,(p-\varepsilon)+1-F(p))=0$. Rearranging terms we obtain $h(p^*)(p^*-\varepsilon)=1$, where p^* is the optimal price set by the pool. Also, we know from the proof of Lemma 2 that when $\sigma=\sigma^*$, the cross-price derivative is zero and the non-cooperative price solves $\sigma = p^e\,h(p^e)$. Replacing this expression in (2) and rearranging we get $h(p^e)(p^e-\varepsilon)=1$, which is the cooperative result. Given that p^e is decreasing in σ, whereas p^* is independent of σ, $p^e > p^*$ when $\sigma < \sigma^*$ and $p^e < p^*$ when $\sigma > \sigma^*$. ■

References

Bergstrom, T. C. (1978): "Cournot equilibrium in factor markets," University of Michigan working paper.

Boldrin, M. and D. K. Levine (2002): "The case against intellectual property," *American Economic Review (Papers and Proceedings)*, 92, 209–212.

Boldrin, M. and D. K. Levine (2005a): "The economics of ideas and intellectual property," *Proceedings of the National Academy of Sciences*, 102, 1252–1256.

Boldrin, M. and D. K. Levine (2005b): "Intellectual property and the efficient allocation of surplus from creation," *Review of Economic Research on Copyright Issues*, 2, 45–66.

Buchanan, J. M. and Y. J. Yoon (2000): "Symmetric tragedies: Commons and anticommons," *Journal of Law and Economics*, 43, 1–13.

Chang, H. F. (1995): "Patent scope, antitrust policy, and cumulative innovation," *RAND Journal of Economics*, 26, 34–57.

Commision on Intellectual Property Rights (2002): "Integrating intellectual property rights and development policy: Report of the commision on intellectual property rights," London.

Cournot, A. (1838): *Researches Into the Mathematical Principles of the Theory of Wealth*, Irwin (1963).

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic competition and optimum product diversity," *American Economic Review*, 67, 297–308.

Ethier, W. J. (1982): "National and international returns to scale in the modern theory of international trade," *American Economic Review*, 72, 389–405.

Graff, G. D., S. E. Cullen, K. J. Bradford, D. Zilberman, and A. B. Bennett (2003): "The public-private structure of intellectual property ownership in agricultural biotechnology," *Nature Biotechnology*, 21, 989–995.

Green, J. R. and S. Scotchmer (1995): "On the division of profit in sequential innovation," *RAND Journal of Economics*, 26, 20–33.

DOI: 10.2202/1935-1682.2863

Heller, M. A. (1998): "The tragedy of the anticommons: Property in the transition from marx to markets," *Harvard Law Review*, 111, 621–688.

Heller, M. A. and R. S. Eisenberg (1998): "Can patents deter innovation? the anticommons in biomedical research," *Science*, 280, 698–701.

Hopenhayn, H., G. Llobet, and M. Mitchell (2006): "Rewarding sequential innovators: Prizes, patents, and buyouts," *Journal of Political Economy*, 114, 1041–1068.

Kim, J. (2004): "What determines aggregate returns to scale?" *Journal of Economic Dynamics and Control*, 28, 1577–1594.

Lerner, J. and J. Tirole (2004): "Efficient patent pools," *American Economic Review*, 94, 691–711.

Llanes, G. and S. Trento (2010): "Patent policy, patent pools, and the accumulation of claims in sequential innovation," *Economic Theory*, available online at http://dx.doi.org/10.1007/s00199-010-0591-5.

Reback, G. L. (2002): "Patently absurd," Forbes.com (June 24), available online at http://www.forbes.com/asap/2002/0624/044.html.

Romer, P. M. (1990): "Endogenous technological change," *Journal of Political Economy*, 98, 71–102.

Scotchmer, S. (1991): "Standing on the shoulders of giants: Cumulative research and the patent law," *Journal of Economic Perspectives*, 5, 29–41.

Scotchmer, S. (1996): "Protecting early innovators: Should second-generation products be patentable?" *RAND Journal of Economics*, 27, 322–331.

Shapiro, C. (2001): "Navigating the patent thicket: Cross licenses, patent pools, and standard setting," in A. B. Jaffe, J. Lerner, and S. Stern, eds., *Innovation policy and the economy*, volume 1, MIT Press for the NBER, 119–150.

Sonnenschein, H. (1968): "The dual of duopoly is complementary monopoly: or, two of cournot's theories are one," *Journal of Political Economy*, 76, 316–318.