

PRIVATE CONTRACTS IN TWO-SIDED MARKETS*

GASTÓN LLANES[†] AND FRANCISCO RUIZ-ALISEDA[‡]

ABSTRACT. We study a platform that connects buyers and sellers. We find that secret contracting implies interrelated hold-up problems for buyers and sellers that reduce platform profits and welfare. By increasing its control over sellers' prices, the platform is able to increase price transparency and commit not to behave opportunistically, which increases platform profits and welfare. Thus, policy prescriptions for dealing with contractual secrecy are reversed in the case of two-sided markets, relative to one-sided markets. Our results can explain the widespread use (and social desirability) of price-forcing contracts, the subscription-based and merchant business models, and integration by platforms.

KEYWORDS: Two-Sided Markets, Platforms, Vertical Relations, Most Favored Nation, Price-Forcing Contracts, Contract Privacy, Agency Model of Sales (JEL: L14, L42).

Date: June 26, 2018.

[†]Pontificia Universidad Católica de Chile, gaston@llanes.com.ar.

[‡]Pontificia Universidad Católica de Chile, f.ruiz-aliseda@uc.cl.

1. INTRODUCTION

Private contracts are common in two-sided markets. For example, Amazon signs private contracts with publishers, Netflix with movie studios, Sony and Nvidia with video game developers, Spotify with record companies, HMOs with healthcare providers, Google with phone manufacturers, Apple with cellphone carriers, and Intel and Microsoft with computer manufacturers.

Previous works studying two-sided markets assume that contracts are publicly observable to all agents (see, e.g., Caillaud and Jullien, 2001, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006a). In this paper, instead, we assume that a platform that connects buyers and (competing) sellers signs a private contract with each seller. Contractual privacy gives rise to interrelated hold up problems for sellers (given that they do not observe the royalties offered to other sellers) and buyers (given that they do not observe sellers' prices –which depend on royalties– when deciding whether to acquire the platform).

Our main contribution is to show that, by increasing its control over sellers' prices, the platform is able to increase price transparency for buyers and sellers and to commit not to behave opportunistically, which increases not only platform profits but also welfare.

The platform may enhance its control over sellers' prices by using price-forcing contracts (iTunes, Uber, Lending Club),¹ by offering a subscription-based business model (Hulu, Spotify, Netflix, PlayStation Plus, HMOs), by becoming a retailer or merchant (Amazon, Zappos), or by integrating with sellers (Netflix, Comcast, HMOs).² These widely used mechanisms allow the platform to overcome the problems caused by contractual secrecy, even if the contracts it signs with sellers are private.³

Our finding that a strict price control by the platform can benefit society stands in stark contrast with the results of papers studying private contracts in one-sided markets (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004). This literature finds that attempts by an upstream provider to increase its control over downstream sellers (e.g., through vertical restraints) are detrimental to welfare. In contrast with what happens in a one-sided market, we show that such actions should not be deemed anticompetitive in a two-sided market.

The above mechanisms increase price transparency for all agents, which allows the platform to address the hold up concerns of buyers and sellers at the same time. However, if the platform cannot solve the informational problems of buyers, an improvement in sellers' information may actually lead to worse outcomes for the platform and society. This result obtains when sellers' goods are substitutes, in which case contractual opportunism with sellers lowers platform

¹Price-forcing contracts are an extreme form of resale price maintenance in which the upper and lower bounds on sellers' prices coincide. For example, Uber leaves no discretion to drivers about the prices charged for their services.

²The platform may also commit not to behave opportunistically by building a reputation. More generally, the platform may combine formal contractual provisions and reputation to achieve commitment.

³Even publicly disclosed contracts may be secretly renegotiated, in which case they are *de facto* private.

royalties and sellers' prices, which in turn mitigates the opportunism with buyers. When sellers' goods are instead complements, contractual opportunism with sellers increases royalties and sellers' prices, which intensifies the hold up problem of buyers.

These results imply that most favored nation (MFN) clauses are desirable only when sellers' goods are complements, given that they improve sellers' information but do not affect buyers' information. This may explain the use of MFN clauses in the private contract between Spotify and Sony –recently made public by North Korean hackers– which is socially and privately optimal if the music portfolios of different labels are complementary.

Our result that there are situations in which the platform may benefit from weakening its price control over sellers because it serves as a commitment device for increasing buyers' surplus is new to the literature, and arises because of the interaction between the two-sidedness of the market and contractual secrecy.

We derive these results by studying a two-period game in which buyers demand platform-based goods from sellers. Sellers' goods may be substitutes or complements. In the first period, the platform provider chooses membership or access fees for buyers and sellers, and sets the royalty fees that sellers have to pay for each unit they sell to buyers; then sellers decide whether to accept the two-part-tariff contract offered by the platform and buyers decide whether to join the platform. In the second period, sellers post prices and buyers with access to the platform choose how much to buy from each seller.⁴

When contracts are not publicly observable, equilibrium behavior depends on how players form beliefs when they observe out-of-equilibrium play. In line with the literature, we assume that buyers form “passive beliefs” (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Hagiu and Hałaburda, 2014) and sellers form “wary beliefs” (McAfee and Schwartz, 1994; Rey and Vergé, 2004).⁵

Even if the main message of the paper is that platform profitability and welfare increase when the informational problems of buyers and sellers disappear, at least three aspects are worth keeping in mind. First, contract privacy is often an environmental constraint for sellers and platforms (e.g., any contract publicly shown can always be secretly renegotiated, so contracts are *de facto* private). Second, our results imply that policies that attempt to remove informational frictions may actually be worse for the platform and buyers if they improve sellers' information but fail to affect buyers' information (e.g., most favored nation clauses are desirable only if sellers' goods are complementary). Third, and this is a key message of the paper, there are ways in which a platform provider can get around the problems arising from informational frictions without making

⁴As a robustness check, we also study a model in which buyers decide to join the platform after observing sellers' prices (see Section 9 for details).

⁵Thus, a buyer who observes an unexpected price from the platform believes that sellers' pricing behavior is unaffected, and a seller observing an unexpected two-part tariff believes that the platform has deviated in a profit-maximizing manner with other sellers. In Section 8, we show that our insights persist when sellers form passive beliefs rather than wary beliefs.

contracts public (e.g., contracting on sellers' prices with both buyers and sellers). Although these practices would be often treated as anticompetitive in a one-sided market, in the current setting they are beneficial not only for the platform but also for buyers.

Our paper contributes to the literature on two-sided markets. To the best of our knowledge, the entire literature assumes that contracts are publicly observable to all parties. The only exception in which one of the two sides does not observe the price charged to the other side is the paper by Hagiu and Hałaburda (2014), which examines how price transparency affects market outcomes. In contrast with our paper, Hagiu and Hałaburda (2014) do not allow for direct transactions between sellers and buyers, which is crucial for our results, and do not study contractual opportunism with both types of players. Another contribution to the two-sided markets literature is that we allow sellers to enjoy market power and study how the platform shapes their competitive interaction through its choice of royalty fees, something that has an effect on platform adoption by buyers.

Our paper also contributes to the literature on vertical relations regulated by secret contracts. Our contribution is to consider a market structure in which an upstream firm has a pricing relationship not only with downstream firms but also with final-good buyers. The two-sidedness of the problem implies that policy conclusions are overturned in comparison with those that result from studying a one-sided market.⁶ Another contribution to this literature is that we study the case in which sellers produce complementary products, which has been unexplored so far.

2. THE MODEL

We study a market that consists of a two-sided platform, $n \geq 2$ sellers, and a continuum of buyers. The platform provider produces a good that enables the interaction between buyers and sellers (e.g., a video console) at a normalized marginal cost of zero. Sellers sell platform-specific products (e.g., video games) to buyers (e.g., gamers) who buy the platform. Sellers produce at zero marginal cost, again a normalization.⁷

Buyers are uniformly spread on the positive real line (with unit density), and the platform is located at the left end. The utility derived by a buyer located at distance $x \in [0, \infty)$ from the platform if she purchases it at price p_0 , and buys $q_i \geq 0$ units of the product of seller $i \in \{1, \dots, n\}$ at price p_i per unit is (Vives, 2001)

$$U_x(p_0, p_1, q_1, \dots, p_n, q_n) = u(p_1, q_1, \dots, p_n, q_n) - x - p_0,$$

⁶See Evans and Schmalensee (2015) for a discussion of normative differences that arise due to two-sidedness in other settings.

⁷If sellers had a constant marginal cost of production $c \in [0, 1)$, the normalization would be as follows: sellers' prices should be interpreted as markups, and equilibrium prices, royalty fees and sellers' sales should be multiplied by the scaling factor $(1 - c)$; in turn, the number of buyers buying the platform should be multiplied by the scaling factor $(1 - c)^2$, whereas the surplus attained by the platform and buyers should be multiplied by the scaling factor $(1 - c)^4$.

where

$$u(p_1, q_1, \dots, p_n, q_n) = \sum_{i=1}^n q_i - \frac{1}{2} \left(\sum_{i=1}^n q_i^2 + \theta \sum_{i=1}^n \sum_{j=1; j \neq i}^n q_i q_j \right) - \sum_{i=1}^n p_i q_i.$$

Parameter $\theta \in (-1, 1)$ captures the degree of complementarity and substitution between sellers' goods. If $\theta < 0$, goods are complements, with their degree of complementarity decreasing with θ . If $\theta = 0$, goods are independent. If $\theta > 0$, goods are substitutes, with their degree of substitution increasing with θ .

We consider a two-period model. In the first period, the platform offers a contract to each seller, and commits to a price p_0 for buyers; then each seller decides whether to accept the corresponding contract, and buyers observe both p_0 and how many sellers have accepted the contract before having to decide whether to buy the platform. In the second period, sellers set prices for their products and buyers decide how many products to buy from each seller. Our timing reflects the fact that buyers use the platform for many periods, during which platform-specific products are continuously being launched. For instance, buyers of a video console often buy it without observing the prices charged for the games they will consume during the lifetime of the console.

Unless otherwise stated, a contract between seller $i \in \{1, \dots, n\}$ and the platform consists of a fixed fee f_i and a per-unit royalty fee w_i .⁸ If seller i accepts the contract and then sells Q_i units to buyers, its total payment to the platform is $f_i + w_i Q_i$. We take n as given, and, for the most part of the paper, assume $n = 2$. We discuss what happens when $n > 2$ and n grows large in Section 5.

In Section 3, we study a two-sided platform with public contracts, so buyers and sellers observe all contracts before making their decisions. In Section 4, we study a two-sided platform with private contracts. We first examine an intermediate situation in which buyers do not observe the contracts offered to sellers, but sellers observe all contracts (the uninformed buyers case). We then examine the private contracts case, in which buyers do not observe the contracts offered to sellers, and each seller only observes the contract it is offered. We assume throughout that p_0 is contractible and is written in the contract offered to any seller.⁹

In Section 3, we seek for symmetric subgame perfect equilibria. In Section 4, we seek for symmetric perfect Bayesian equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed. Section 8 shows that results are robust to alternative ways of forming out-of-equilibrium-path beliefs.

⁸Therefore, the price at which seller i should sell its goods to buyers is not included in the contract. In Section 7 we study what happens if price-forcing contracts are allowed.

⁹In most occasions, p_0 can be contracted upon. Even if p_0 is not contractible, reputational concerns may prevent the platform from behaving opportunistically with sellers. That p_0 is known by sellers when they have to decide whether to accept contracts is standard in some industries such as video games (Hagiu, 2006a). If p_0 was chosen after sellers have decided to accept the platform's offers, sellers would anticipate a hold up problem that would harm the platform. Note also that it is in principle easier to contract upon p_0 than upon other seller's fees because sellers eventually observe p_0 , but they may never observe royalty fees paid by other sellers.

3. PUBLIC CONTRACTS

In this section, we analyze the case in which contracts are public, which serves as a benchmark for the two other informational scenarios we shall consider because there is no opportunism and the platform provider can fully control sellers' prices through the royalty fees.

We start by studying the second period. After observing p_i and p_j ($i, j \in \{1, 2\}; i \neq j$), buyers who have purchased the platform decide how many units of sellers' products to consume. Looking at interior solutions of a buyer's utility maximization problem yields the following *per capita* demand (conditional on purchase of the platform) for the product of seller i :

$$q_i(p_i, p_j) = \frac{1 - \theta - p_i + \theta p_j}{1 - \theta^2}. \quad (1)$$

Per capita consumption does not depend on the distance between the buyer and the platform. Thus, the overall demand for seller i 's product is $Q_i(p_i, p_j) = x_0 q_i(p_i, p_j)$, where x_0 is the number of buyers who choose to buy the platform in the first period. Seller $i \in \{1, 2\}$ solves the following problem given a price p_j by the other seller:

$$\max_{p_i} \{(p_i - w_i) Q_i(p_i, p_j) - f_i\},$$

where f_i is a cost already sunk and the total number of buyers, x_0 , is given from the first period. Seller i 's first-order condition is

$$x_0(1 - \theta - 2p_i + w_i + \theta p_j) = 0,$$

so its equilibrium price can be easily shown to be

$$p_i(w_i, w_j) = \frac{(2 + \theta)(1 - \theta) + 2w_i + \theta w_j}{(2 + \theta)(2 - \theta)}. \quad (2)$$

It readily follows from (1) that each buyer buys

$$q_i(w_i, w_j) = \frac{(1 - \theta)(2 + \theta) - w_i(2 - \theta^2) + \theta w_j}{(1 - \theta^2)(4 - \theta^2)} \quad (3)$$

units of product i .

Turning to the analysis of the first period, expression (1) implies that buyer x 's utility given p_0 , p_1 and p_2 is

$$U_x(p_1, p_2, p_0) = u(p_1, p_2) - x - p_0,$$

where

$$u(p_1, p_2) = \frac{2(1 - \theta)(1 - p_1)(1 - p_2) + (p_1 - p_2)^2}{2(1 + \theta)(1 - \theta)}.$$

Because of symmetry, it is not very hard to show that optimal royalties must be such that $w_1 = w_2 = w$, so

$$u(w) \equiv u(p_1(w, w), p_2(w, w)) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2}$$

is the utility that any buyer expects to derive from consuming the goods sold by sellers. This results in a demand for the platform equal to

$$x_0(w, p_0) = u(w) - p_0.$$

Anticipating how play will evolve in the second period, seller i will accept the contract offered by the platform if and only if the fixed fee does not exceed the profit it expects to make given the royalty it must pay and the extent of adoption of the platform by buyers. Using symmetry again, it follows that the platform clearly charges fee $f = x_0(w, p_0)q_i(w, w)[p_i(w, w) - w]$ to seller i , so it solves

$$\max_{w, p_0} \{ x_0(w, p_0) [p_0 + p_1(w, w) q_1(w, w) + p_2(w, w) q_2(w, w)] \}. \quad (4)$$

It is straightforward to prove the following result.

Proposition 1 (Public contracts). *If contracts are publicly observed by all parties, equilibrium royalties are $\bar{w} = -(1 - \theta) < 0$, the equilibrium price charged by seller $i \in \{1, 2\}$ is $\bar{p}_i = 0$, the equilibrium price for the platform is $\bar{p}_0 = \frac{1}{2(1+\theta)} > 0$, per capita consumption of each product is $\bar{q}_i = \frac{1}{1+\theta}$, the number of buyers who join the platform is $\bar{x}_0 = \frac{1}{2(1+\theta)}$, platform profits are $\bar{\pi}_0 = \frac{1}{4(1+\theta)^2}$, and consumer surplus is $\bar{cs} = \frac{1}{8(1+\theta)^2}$.*

The optimal royalty fee is always negative and goes to zero as θ goes to one. The platform extracts all the surplus from sellers through the fixed fee. Thus, the platform acts as if it has two instruments to charge buyers: directly through the access fee p_0 and indirectly through sellers' prices. Charging buyers through the access fee is more efficient because seller prices affect buyers' per capita demands and membership decisions, whereas the access fee does not distort consumption of sellers' products and only affects membership decisions. Thus, the platform chooses a negative royalty to induce sellers to choose prices equal to marginal costs (which means that the platform captures zero profits from the seller side), and obtains positive profits through the access fee.¹⁰ As $\theta \rightarrow 1$ (goods become perfect substitutes), prices converge to marginal cost due to pure

¹⁰Formally, the first-order condition of the platform with respect to price p_0 is

$$x_0 + \frac{\partial x_0}{\partial p_0} (p_0 + p_1 q_1 + p_2 q_2) = 0.$$

Given that the platform perfectly anticipates second-period prices as a function of royalties, it can solve the problem in expression (4) as if it was choosing prices p_i instead of royalties w_i . Then it would choose price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) + \frac{\partial x_0}{\partial p_1} (p_0 + p_1 q_1 + p_2 q_2) = 0. \quad (5)$$

The first-order condition with respect to p_0 implies that $p_0 + p_1 q_1 + p_2 q_2 = x_0$, given that $\partial x_0 / \partial p_0 = -1$. By Roy's identity, $\partial x_0 / \partial p_1 = -q_1$, so the first-order condition in (5) becomes

$$x_0 \left(p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

Bertrand competition, so the platform does not have to subsidize sellers to induce efficient downstream pricing, and the equilibrium royalty converges to zero. Hence the results in Proposition 1.

Finally, note that the results in this section stand in stark contrast with those of the vertical relations literature (which focuses on one-sided markets). In a one-sided market, the upstream producer (platform) would choose royalties to maximize seller's joint revenues, which would lead to positive markups. Our results differ because of the two-sidedness embedded in the model.

4. PRIVATE CONTRACTS

In this section, we assume that the contracts between the platform and sellers are private. Thus, buyers cannot observe any of the contracts offered to sellers, and a seller cannot observe the contract offered to the other seller. We will seek for symmetric Perfect Bayesian Equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.¹¹ In what follows, let p_0^* denote the price charged to buyers in a symmetric PBE. Also, let w^* denote the royalty fee that is offered to seller $i \in \{1, 2\}$ in a symmetric PBE, and f^* the associated fixed fee.

Regarding the formation of out-of-equilibrium beliefs, note that, upon observing any $p_0 \neq p_0^*$, rational buyers would realize that such a deviation affects sellers' profits and potentially their incentives to enter the market (this happens when $p_0 > p_0^*$). They should therefore conclude that a price deviation must be accompanied by a change in the fixed fee and/or a change in the royalty fee offered to each seller. We will look at equilibria in which buyers rationalize any price deviation by conjecturing that there was no deviation in the royalty fee offered to each seller; hence, buyers believe upon observing $p_0 \neq p_0^*$ that the platform is simply adjusting the fixed fee offered to each seller just to make it break-even given w^* . These beliefs are in the spirit of "passive beliefs" (Hart and Tirole, 1990), but they require some rationality by buyers. In particular, when buyers observe a price deviation, they acknowledge that this should have had an impact on the sellers' willingness to accept the contract, and they reason that the absence of such an impact must be due to a change in the fixed fee offered to each seller. We refer to this weak form of passive beliefs held by buyers as "weakly passive beliefs," and note that the main implication of

In a symmetric equilibrium,

$$x_0 p_1 \left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) = 0,$$

so it is optimal to induce sellers to sell their products at their marginal cost of zero. Given that seller 1 chooses price p_1 so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0,$$

the royalty fee must be negative so that sellers choose prices equal to their marginal costs.

¹¹No asymmetric equilibrium exists, so the symmetry requirement is without loss of generality, at least if one restricts attention to equilibria in which the pricing strategy and beliefs held by a seller are polynomial functions of the royalties it observes.

such belief formation is that buyers always expect the interaction of sellers in the product market to be unaffected by the choice of p_0 .¹²

Because a seller anticipates such unsophisticated behavior by buyers when $p_0 \neq p_0^*$, it believes that $p_0 \neq p_0^*$ conveys no information about contract offers. Thus, sellers form passive beliefs with respect to deviations in p_0 . On the other hand, seller $i \in \{1, 2\}$ is assumed to form “wary beliefs” (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when it observes an unexpected contract offer $(w_i, f_i) \neq (w^*, f^*)$. In such case, it believes that the platform must have made an offer to j that maximizes the platform’s total profit given the price that it charges to buyers and the contract offered to seller i . We assume as well that seller i conjectures that the other seller forms wary beliefs, and that the platform does not want to drive the other seller out of the market. We emphasize that, in equilibrium, a seller perfectly anticipates the offer made by the platform to the other seller, as is usual. In turn, buyers also anticipate perfectly the contract offers made to sellers in equilibrium.

4.1. Uninformed buyers. Before examining equilibrium play when the contract offer received by a seller is solely observed by this seller, it is useful to examine an intermediate case in which sellers observe each other’s contract, but buyers do not. As we show next, this unobservability gives rise to a hold up problem: buyers (correctly) anticipate that the platform is inducing sellers to charge high (collusive) prices. Collusive pricing by sellers implies that they will earn more for each buyer who joins the platform, but the platform’s value to buyers will be harmed by such beliefs. Both these forces induce the platform to lower access prices for buyers and thus entice them to buy the platform. A revealed preference argument then readily implies that platform profits decrease relative to when contracts are public, whereas consumer surplus also decreases, since fewer buyers join the platform and the per capita surplus of those joining is lower.

In what follows, let (\hat{f}, \hat{w}) denote the contract offered to each seller in equilibrium. Because buyers cannot observe deviations from this contract and form weakly passive beliefs when observing any p_0 , their demand for the platform is

$$x_0(\hat{w}, p_0) = \frac{(1 - \hat{w})^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

The platform extracts all the surplus from sellers, and chooses p_0 , w_1 and w_2 to maximize

$$x_0(\hat{w}, p_0) [p_0 + p_1(w_1, w_2)q_1(w_1, w_2) + p_2(w_2, w_1)q_2(w_2, w_1)], \quad (6)$$

¹²The outcome would be the same under the standard strong form of passive beliefs (that is, buyers do not change their equilibrium beliefs when observing out-of-equilibrium behavior). However, it would be harder to interpret some situations. For example, upon observing $p_0 > p_0^*$, a buyer who kept her beliefs about f^* and w^* should conclude that sellers are accepting contracts that lead to negative profits, for buyer demand is smaller than it should be in equilibrium (below, we show that buyer demand for the platform does not affect competition between sellers, which solely depends on royalty fees).

where $p_i(w_i, w_j)$ and $q_i(w_i, w_j)$ are given by expressions (2) and (3). The first-order condition corresponding to w_i is

$$\frac{\theta(1-\theta)(2+\theta)^2 - (8-6\theta^2)w_i + 2\theta^3 w_j}{(1-\theta^2)(4-\theta^2)^2} = 0.$$

Rearranging this equation allows us to give it an interpretation that will be useful later on. If seller $j \neq i$ knows that seller i can observe all contract offers, there is no need for seller j to observe the royalty fee offered by the platform provider to the other seller. The point is that, upon observing $w_j = w$, seller j can rationally infer that the platform provider finds it optimal to charge seller i with a royalty fee w_i that solves the above first-order condition. Thus, given a royalty offer of w , a seller believes that the other seller is being offered a royalty equal to

$$\widehat{w}^*(w) = \frac{\theta(1-\theta)(2+\theta)^2 + 2\theta^3 w}{2(4-3\theta^2)}. \quad (7)$$

Therefore, $\widehat{w}^*(w)$ can be interpreted as seller j 's belief about the royalty fee offered to the other seller when seller j observes w . Note that such a belief is correct both on and off the equilibrium path because it follows from optimal behavior by the platform provider for *any* w . The function $\widehat{w}^*(\cdot)$ will serve as a useful benchmark when we further assume in the next section that sellers cannot observe each other's contract offers.

Solving for the equilibrium royalty fee yields the following proposition.

Proposition 2 (Uninformed buyers). *If sellers observe all contracts but buyers do not observe sellers' contracts with the platform, equilibrium royalties are $\widehat{w} = \frac{\theta}{2}$, the equilibrium price charged by seller $i \in \{1, 2\}$ is $\widehat{p}_i = \frac{1}{2} > 0$, the equilibrium price for the platform is $\widehat{p}_0 = -\frac{1}{8(1+\theta)} < 0$, per capita consumption of each product is $\widehat{q}_i = \frac{1}{2(1+\theta)}$, the number of buyers who join the platform is $\widehat{x}_0 = \frac{3}{8(1+\theta)}$, platform profits are $\widehat{\pi}_0 = \frac{9}{64(1+\theta)^2}$, and consumer surplus is $\widehat{cs} = \frac{9}{128(1+\theta)^2}$.*

Proposition 2 shows that the royalty fee is positive if $\theta > 0$ and negative if $\theta < 0$. When buyers do not observe sellers' contracts, they become unresponsive to changes in the royalty fee. As a result, the platform chooses royalties to maximize seller profits, taking as given the number of buyers. If sellers' goods are substitutes (complements), the platform sets positive (negative) royalties, to make sellers internalize the positive (negative) cross-price effects between their demands.¹³

¹³Formally, if the platform acts as if it is choosing price p_1 instead of royalty fee w_1 , it chooses price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

This first-order condition differs from the one in Section 3 (footnote 10) because buyers do not observe changes in royalty fees, so their decision to buy the platform depends only on their *beliefs* about the equilibrium royalty. In a symmetric equilibrium, it holds that

$$-\left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) p_i = q_i,$$

Given \hat{p}_i and \hat{q}_i , expression (6) yields that the optimal access price for buyers solves

$$\hat{p}_0 = \frac{u(\hat{p}_1, \hat{p}_2) - (\hat{p}_1 \hat{q}_1 + \hat{p}_2 \hat{q}_2)}{2}.$$

In comparison with the public contracts case, the platform has incentives to lower p_0 for two reasons: (i) because consumer surplus from consumption of seller goods decreases ($u(\hat{p}_1, \hat{p}_2) < u(0, 0)$), and (ii) because seller surplus per buyer increases ($\hat{p}_1 \hat{q}_1 + \hat{p}_2 \hat{q}_2 > 0$), so each additional buyer becomes more valuable for the platform.¹⁴

In the previous section, the (observable) royalty fees gave both control over sellers' prices and commitment vis-à-vis buyers. In the current section, the royalties retained their values as instruments to control sellers' prices, but their lack of observability by buyers made it impossible for the platform to commit not to act opportunistically vis-à-vis buyers. The next section examines what happens if the unobservability of royalty fees by buyers is accompanied by a loss in the control over sellers' prices due to each seller not observing the other seller's royalty.

4.2. Private contracts. We now turn to the analysis of private contracts, in which the contract offered to a seller is solely observed by this seller. At this point, one may be tempted to extrapolate Rey and Vergé's (2004) finding that the platform must be worse off (relative to the uninformed buyers case) for it loses control over sellers' prices, at least when sellers produce substitutes, the setting considered in Rey and Vergé (2004). This section shows that such an extrapolation would be incorrect as it would miss the feedback loops that arise in a two-sided market. When goods are substitutes, we shall see that weakening the platform's control over sellers' prices acts as a commitment device for raising buyers' surplus. The point is that sellers fear that the platform will behave opportunistically, offering lower royalties to other sellers when they accept their contract, so sellers are willing to accept lower royalties from the platform. This decreases royalties and sellers' prices relative to the uninformed buyers case. The fear of opportunism of sellers effectively makes the platform lose part of its control over sellers' prices, but offsets the fear that buyers have that the platform will behave opportunistically with them, so the foreseen decrease in sellers' prices encourages buyers to join the platform. As a result, the platform can charge higher access prices to buyers and still increase the number of buyers joining the platform. These effects dominate the smaller profit per buyer that can be extracted from sellers, so platform profits increase relative to the uninformed buyers case when sellers' goods are substitutes. When such goods are complements, we shall show that the converse holds: the lack of commitment

so the optimal implied price for sellers is positive. Finally, seller i chooses price p_i so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0.$$

Thus, the royalty needs to be positive if the cross-price effect $\partial q_2 / \partial p_1$ is positive (substitutes), and negative if the cross-price effect is negative (complements).

¹⁴In the case at hand, it turns out that the platform lowers p_0 so much that it ends up setting a negative access fee for buyers, but p_0 may be positive for other specifications of demand.

when setting sellers' royalties acts as a commitment device for inducing higher prices by sellers, which accentuates rather than mitigates the hold up problem borne by buyers.

Turning to the formal analysis, note that at the beginning of the second period, seller $i \in \{1, 2\}$ observes p_0 , x_0 , f_i and w_i , and chooses a price for its product based on this information. Let $Q_i(p_i, p_j) = x_0 q_i(p_i, p_j)$ be seller i 's overall demand and let $B(w_i)$ denote the belief formed by seller i about the royalty fee paid by seller j to the platform when seller i observes a contract with royalty fee w_i .¹⁵ We follow Rey and Vergé (2004), and restrict attention to equilibria in which seller i 's belief about the royalty fee paid by the other seller does not depend on the fixed fee it observes. Not only is the pricing strategy of seller i independent from the fixed fee it already paid, but it is also independent from p_0 (and hence from x_0). Such a price has no signaling role and does not affect belief formation, which seems a reasonable assumption given that x_0 is simply a scaling factor in seller i 's second-period profit.¹⁶

Let $p_i(w_i)$ denote the strategy of seller $i \in \{1, 2\}$ in the second-period subgame if it has observed an offer of (w_i, f_i) and price p_0 . Seller $i \in \{1, 2\}$ chooses p_i to maximize

$$(p_i - w_i) Q_i(p_i, p_j(B(w_i))) - f_i,$$

with f_i already sunk. The first-order condition is

$$1 - \theta + w_i - 2p_i(w_i) + \theta p_j(B(w_i)) = 0. \quad (8)$$

We now turn to analyzing the first period of play. Regardless of the price p_0 that buyers observe, they believe that both sellers face a royalty fee w^* , so they expect a price

$$p_i^* = \frac{1 - \theta + w^*}{2 - \theta}$$

for each unit they purchase from seller $i \in \{1, 2\}$ in the second period. Given price p_0 , the overall utility expected by buyer x is

$$U_x(w^*, p_0) = \frac{(1 - w^*)^2}{(1 + \theta)(2 - \theta)^2} - x - p_0,$$

so the demand for the platform is

$$x_0(w^*, p_0) = \frac{(1 - w^*)^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

¹⁵Because we are looking at symmetric equilibria, the belief function $B(\cdot)$ does not depend on the label of the seller receiving the possibly unexpected offer. In general, $B(\cdot)$ is an unrestricted function except for the constraint that $B(w^*) = w^*$ (i.e., conjectured beliefs are fulfilled along the equilibrium path). In this section, we further restrict the function so that beliefs are wary. In Section 8, we study what happens if beliefs are passive, so that $B(w) = w^*$ for $w \neq w^*$.

¹⁶Therefore, it does not affect equilibrium pricing in the second period if sellers believe that it does not convey some information, making it self-fulfilling that it is pointless for the platform to use it for signaling purposes.

The platform's total profit if it charges p_0 and makes a private offer of (w_1, f_1) and (w_2, f_2) to sellers 1 and 2 is

$$\pi_0(w_1, f_1, w_2, f_2, p_0) = x_0(w^*, p_0)[p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1))] + f_1 + f_2,$$

since the platform can perfectly anticipate actual sales made by sellers 1 and 2. In order for seller 2 (say) to form wary beliefs, her inference about seller 1's contract upon observing a price of p_0 and an offer of (w_2, f_2) must be such that $B(w_2)$ maximizes $\pi_0(w, f, w_2, f_2, p_0)$ with respect to w and f subject to the constraint that

$$f \leq (p_1(w) - w)x_0(w^*, p_0)q_1(p_1(w), p_2(B(w))).$$

Taking into account that the constraint must bind at the optimum, and that (8) implies

$$q_1(p_1(w), p_2(B(w))) = \frac{p_1(w) - w}{1 - \theta^2},$$

we have

$$B(w_2) \in \underset{w}{\operatorname{argmax}} \check{\pi}_0(w, w_2, f_2, p_0), \quad (9)$$

where

$$\begin{aligned} \check{\pi}_0(w, w_2, f_2, p_0) &= x_0(w^*, p_0) \left\{ p_0 + w q_1(p_1(w), p_2(w_2)) \right. \\ &\quad \left. + w_2 q_2(p_2(w_2), p_1(w)) + \frac{[p_1(w) - w]^2}{1 - \theta^2} \right\} + f_2. \end{aligned}$$

The first-order condition with respect to w is

$$\begin{aligned} & q_1(p_1(w), p_2(w_2)) + \frac{2[p_1(w) - w]}{1 - \theta^2} \left(\frac{dp_1(w)}{dw} - 1 \right) \\ & + \left[w \frac{\partial q_1(p_1(w), p_2(w_2))}{\partial p_1} + w_2 \frac{\partial q_2(p_2(w_2), p_1(w))}{\partial p_1} \right] \frac{dp_1(w)}{dw} = 0. \end{aligned} \quad (10)$$

Evaluating this expression at $w = B(w_2)$, and letting $p_i(w) = p(w)$ because of symmetry, we obtain

$$\begin{aligned} & 1 - \theta - p(B(w_2)) + \theta p(w_2) \\ & + (\theta w_2 - B(w_2)) \frac{dp(B(w_2))}{dw} + 2[p(B(w_2)) - B(w_2)] \left[\frac{dp(B(w_2))}{dw} - 1 \right] = 0. \end{aligned} \quad (11)$$

If one focuses on PBE such that $p(\cdot)$ and $B(\cdot)$ are polynomial functions, then Rey and Vergé (2004) show that there is no loss of generality in restricting attention to affine functions, so one can readily solve the system of differential equations given by (11) and (8) (after dropping subscripts). In Appendix A, we prove the following result.

Proposition 3 (Private contracts). *The unique symmetric PBE in which $p(w)$ and $B(w)$ are polynomial functions is such that $p(w) = \Theta_\theta + \Sigma_\theta w$ and $B(w) = \Gamma_\theta + \Phi_\theta w$ for constants $\Theta_\theta \in [0, 1]$,*

$\Sigma_\theta \in [\frac{1}{2}, 1] > 0$, $\Gamma_\theta \in [0, 1]$, and $\Phi_\theta \in [-1, 1]$. In such an equilibrium, it always holds that

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \geq 0$$

for $i \in \{1, 2\}$ as well as that $p_0^* < 0$ and $w^* \geq 0$ for any $\theta \in (-1, 1)$, with $w^* = 0$ if and only if $\theta = 0$. Platform profits are

$$\pi_0^* = \left(\frac{1 - p_i^{*2}}{2(1 + \theta)} \right)^2,$$

and consumer surplus is

$$cs^* = \frac{1}{2} \left(\frac{1 - p_i^{*2}}{2(1 + \theta)} \right)^2.$$

Relative to when sellers can observe each other's contract, it holds when they cannot that the platform loses part of its control over sellers' prices because of its opportunistic behavior when dealing with each on a one-on-one basis (as in Rey and Vergé, 2004). This loss of control implies that the platform cannot sufficiently raise sellers' prices through the royalty fees when goods are substitutes, and that it cannot sufficiently lower these prices when goods are complements. Thus, equilibrium prices decrease in the substitutes case and increase in the complements case, relative to the case in which sellers (but not buyers) can observe all contracts.

More specifically, when buyers cannot observe the contracts offered to sellers, but sellers can, an increase in the royalty fee offered to a seller induces it to price higher, so the platform leans towards raising this seller's sales and thus make greater profit. As readily follows from an inspection of expression (7), this is accomplished by simultaneously offering a greater royalty fee to the other seller if $\theta > 0$ (sellers' prices are strategic complements) and a lower royalty fee if $\theta < 0$ (sellers' prices are strategic substitutes). The same effect, except that it is exacerbated, is into play in the private contracting case because of the possibility that the platform provider acts opportunistically with respect to sellers, since $dB(w)/dw$ can be shown to be positive if $\theta > 0$ and negative if $\theta < 0$.¹⁷ Figure 1 plots $d\widehat{w}^*(w)/dw$ (see solid curve) relative to $dB(w)/dw$ (see dashed curve) as parameter θ varies, illustrating how beliefs become more sensitive to changes in the offer received from the platform when sellers cannot observe each other's contracts.

The properties of $dB(w)/dw$ are instrumental in understanding the properties of w^* highlighted in Proposition 3. Contrary to the case in which sellers can observe each other's royalties (see discussion in Section 4.1), royalty fees are never negative under private contracting, regardless

¹⁷This can be seen formally from expression (10): by the implicit function theorem, it holds that

$$\frac{dB(w_2)}{dw_2} = - \frac{\theta \left(\frac{dp_2(w_2)}{dw_2} + \frac{dp_1(w)}{dw} \right)}{\partial^2 \pi_0'(w, w_2, f_2, p_0) / \partial w^2},$$

so the strict concavity of $\pi_0'(w, w_2, f_2, p_0)$ with respect to w , together with symmetry and the fact that $dp(w)/dw > 0$, yields that the sign of $dB(w)/dw$ coincides with that of θ .

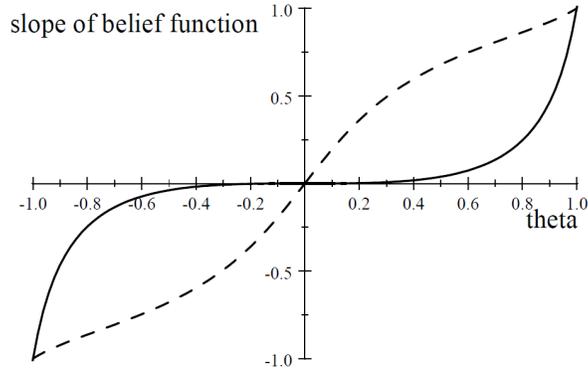


FIGURE 1. Comparison of beliefs in the uninformed buyers (solid curve) and private contracts (dashed curve) cases

of the value of θ . As we show in Appendix B, w^* is nonnegative if and only if an increase in w_1 induces seller 1 to believe it will sell more because of the conjectured change in the other seller's price. Regardless of the sign of θ , it always holds that an increase in w_1 has this effect on seller 1. The point is that a greater w_1 induces seller 1 to price higher, and seller 1 always believes that the platform can better profit from such price increase by increasing this seller's sales,¹⁸ which explains why w^* is always nonnegative, unlike \widehat{w} . Figure 2 compares royalty fees in the two models (the dashed curve corresponds to the case of private contracting).

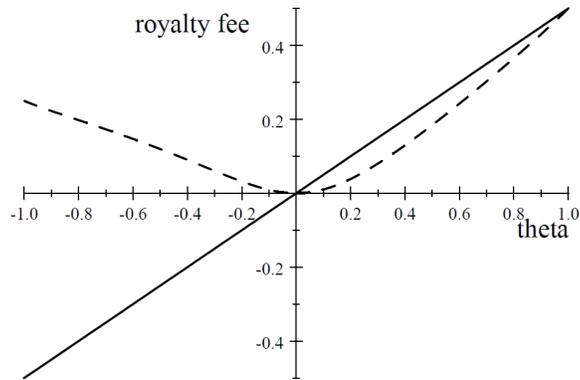


FIGURE 2. Comparison of royalty fees in the uninformed buyers (solid curve) and private contracts (dashed curve) cases

Because $w^* < \widehat{w}$ if and only if $\theta > 0$, it should come as no surprise that the comparison of sellers' prices in both situations is as illustrated in Figure 3 (the dashed curve corresponds to the case of private contracting).

¹⁸When goods are substitutes, the platform is interested in making seller 2 raise its price; when goods are complements, the platform is instead interested in making seller 2 lower its price. Either way, seller 1 believes it will make greater sales following an increase in w_1 because of the conjectured price change that seller 2 is induced to perform by the platform provider.

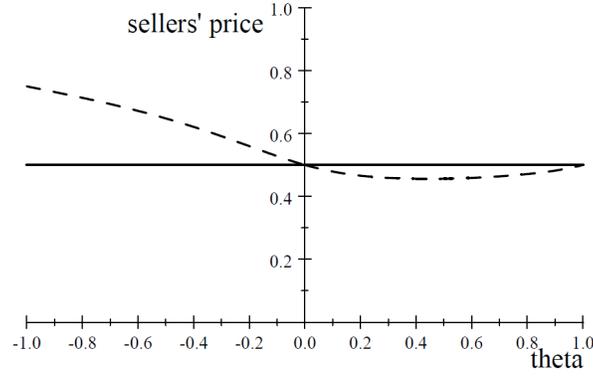


FIGURE 3. Comparison of seller prices in the uninformed buyers (solid curve) and private contracts (dashed curve) cases

The platform's lower control on pricing by sellers when they cannot observe each other's contracts has key implications for the optimal access price for buyers. Given p_i^* and q_i^* , the optimal access price is

$$p_0^* = \frac{u(p_1^*, p_2^*) - (p_1^* q_1^* + p_2^* q_2^*)}{2},$$

so let us first pay attention to the buyer utility determinant of the optimal access price for buyers. When sellers' goods are complements, buyers correctly anticipate that sellers will charge higher prices when they cannot observe each other's offer than when they can, so the platform has an incentive to lower the platform's price relative to when sellers can observe each other's offer. When sellers' goods are substitutes, sellers charge lower prices when they cannot observe each other's offer than when they can, so the platform has an incentive to raise the platform's price relative to when sellers can observe each other's offer.

The other determinant of the optimal access price for buyers is how much overall profit is generated per buyer through the two sellers. Figure 4 shows how total profit generated by sellers per customer varies with θ (the dashed curve represents the situation when seller cannot observe each other's offer).

Because sellers are induced to price collusively when they can observe each other's offer, it holds that per-buyer profitability is at least as large as when they cannot observe each other's offer. This implies that, regardless of the value of θ , the platform has an incentive to set a higher price for the platform when sellers cannot observe each other's offer than when they can. Interestingly, note that the incentive is very small when $\theta > 0$: in such cases, the platform's opportunistic behavior is hardly costly in terms of generating sellers' profits. The effect highlighted by Rey and Vergé (2004) is present, but it is not very strong.

Overall, we find that pricing by the platform is mostly driven by the anticipated effect of sellers' prices on buyer utility. On the one hand, when $\theta > 0$, the platform prices higher when sellers cannot observe each other's offer than when they can: the effect on buyer demand of having

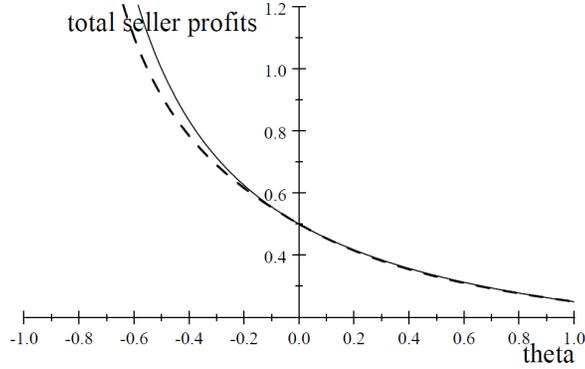


FIGURE 4. Comparison of seller profits per customer in the uninformed buyers (solid curve) and private contracts (dashed curve) cases

lower prices dominates the effect of appropriating less profit through sellers. On the other hand, when $\theta < 0$, the effect of having lower buyer utility when sellers cannot observe each other's offer always dominates the lower per-buyer profitability that arises when sellers cannot observe each other's offer. This is illustrated by Figure 5 (the dashed curve corresponds to the case of private contracting).

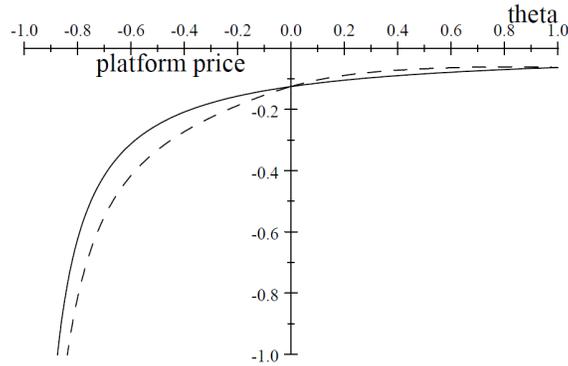


FIGURE 5. Comparison of buyer access prices in the uninformed buyers (solid curve) and private contracts (dashed curve) cases

It should then not be very surprising that platform profits are greater when sellers cannot observe each other's offer than when they can if and only if $\theta > 0$, as Figure 6 shows (the dashed curve represents the situation when seller cannot observe each other's offer). A similar result holds for buyer and total welfare, since they are proportional to platform profits both if sellers can or cannot observe each other's offer.

We now turn to our main result regarding the effect of private contracting on price structures, profits and welfare. The following proposition shows the effects of private contracts in a two-sided platform by comparing the equilibrium of this section with that of the previous two sections.

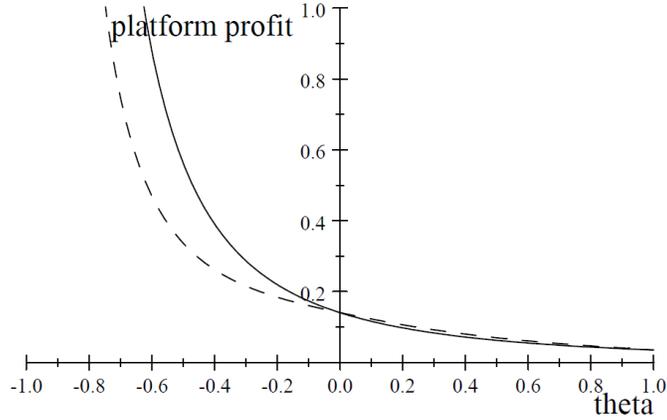


FIGURE 6. Comparison of platform profit

Proposition 4 (Private vs. public contracts). *Equilibrium royalties are negative with public contracts, and are positive with private contracts. The price of the platform for buyers is positive with public contracts, and is negative with private contracts. Private contracts lead to lower profit, consumer surplus, and welfare.*

The first two claims in the proposition follow from comparing the equilibria of the models in Sections 3 and 4. The proof for the last claim is included in the proof of Proposition 3.

5. PRIVATE CONTRACTS WITH A LARGE NUMBER OF SELLERS

An interesting case to examine is that in which n becomes very large and strategic interaction among sellers becomes very weak.

If $\theta > 0$, sellers compete monopolistically and the platform ends up having an absolute control over sellers' prices through royalty fees. As $n \rightarrow \infty$, both $p(w)$ and $B(w)$ converge pointwise to w , whereas w^* and p^* both converge to $1/2$ for $\theta > 0$.¹⁹

The platform therefore has no incentive to act opportunistically with respect to any seller and it induces collusive pricing by sellers in equilibrium, thus depressing buyers' incentives to join the platform as in Section 4.1.

This result gives a rationale for the platform to limit entry by sellers: when the platform can act opportunistically, it prefers that there be strategic interaction among sellers so as to commit to control less their pricing and thus alleviate the buyers' hold up problem. Restricting massive

¹⁹These arguments can be shown formally and the proof is available upon request. We note that $p(w) = (1 + w)/2$ and $B(w) = 0$ when $\theta = 0$, so $w^* = 0$ and $p^* = 1/2$ when sellers' products are independent. Therefore, there is a discontinuity at $\theta = 0$ in terms of pricing strategies, belief functions and royalties, but not in terms of the price that sellers are induced to charge.

market participation by sellers would not only benefit the platform but also buyers, so it would be welfare-enhancing if there are no love-for-variety effects.²⁰

6. FREE PLATFORMS

Another situation that one sometimes observes in the real world is that in which the platform is free for buyers ($p_0 = 0$ by assumption).²¹ By a standard revealed-preference argument, the platform earns less in each of the scenarios we have contemplated. It is also intuitive that having $p_0 = 0$ only changes platform pricing on the seller side when contracts are public, since this is the only scenario in which the provider was tilting towards making money on the buyer side.

When contracts are public, having $p_0 = 0$ forces the platform to make money on the seller side and results in higher prices charged by sellers because of the higher royalty fees they face. When buyers cannot observe the contracts offered to sellers, sellers' prices (and royalty fees) are unaffected by having $p_0 = 0$ regardless of whether or not sellers can observe each other's contract. This is a natural result because in these situations buyers' demand depends on their beliefs about the prices that sellers will charge in the product market, and such prices are independent from the access price paid by buyers.

Even though we do not perform the analysis for the sake of space constraints, the qualitative results regarding profits and welfare when p_0 can be chosen by the platform provider carry over when p_0 is restricted to equal 0 (proof available on request). Our findings therefore do not depend on such aspects.

7. POLICY IMPLICATIONS

Thus far, we have shown that private contracting creates several hold up problems that are interrelated: the hold up problem faced by each seller because it does not observe the other seller's royalty mitigates (reinforces) the hold up problem faced by buyers when sellers sell substitute (complementary) goods. Overall, the informational problem faced by buyers is the main driver of our results, so this section deals with some alternative policies that may help the platform to solve the buyers' hold up problem. The principle that underlies these policies is to provide guarantees by the platform that buyers will not be held up after purchasing the platform.

Conceptually, we argue that the platform aims at directly controlling pricing of the platform-based goods, so that it can then commit to the pricing studied in the benchmark case of public contracts. There are ways to commit in the absence of explicit contracts, namely, repeated interaction among patient enough parties. Rather than focusing on these well-known mechanisms

²⁰Our result that a platform may find it optimal to restrict access is different from Halaburda, Piskorski, and Yildirim (forthcoming), who show that restricting access may increase the valuation of one side of the market by reducing competition from other agents on that side, and apply this analysis to matching platforms such as eHarmony.

²¹See Hagiu (2006b) and Casadesus-Masanell and Llanes (2015) for analyses of free (open) platforms.

that are likely to be relevant in the real world, we focus instead on some practices that are frequently used and may appear as anticompetitive in light of past research on one-sided markets. We remark that we do not view all the possible mechanisms to achieve both direct price control and commitment to transparent pricing as exclusive, but we will examine each separately, not paying any attention to their interaction.

7.1. Price-forcing clauses. A natural way for the platform provider to assure buyers are not held up after acquiring the platform is to add a price-forcing clause to the contracts signed with each seller and with buyers (as in the case of iTunes). The provider of the platform can force sellers to accept selling their goods at marginal cost as in Section 3, and then include this price commitment when contracting with buyers buying the platform.²² Even if contracts signed with sellers remain private, all the hold up problems disappear and the platform can replicate the public contracts outcome, thus enhancing its profitability. Because consumer surplus also increases, we obtain our main finding that a dominant platform that can enhance its control over sellers' prices increases social welfare.

Proposition 5 (Price-forcing contracts). *Relative to when price-forcing clauses are not used, the introduction of such clauses always increases total welfare.*

Price-forcing contracts are advantageous to the platform and to buyers as well because they allow the platform to commit to low seller prices and stimulate platform adoption. This result may explain why price-forcing contracts are common in two-sided platforms, as in the cases of iTunes and Uber. In the case of Sony's PlayStation Plus platform, for example, gamers pay a yearly fee for access to free or discounted games. Developers negotiate PS Plus contracts with Sony on a case-by-case basis, and agree on the price at which they will sell games on the platform. Sony, in turn, commits to having a large number of games at zero or discounted price available for PS Plus' subscribers, which provides insurance to buyers about future game prices.²³

To the best of our knowledge, ours is the first paper providing a rationale for the use of price-forcing clauses in a two-sided platform setting. Use of such clauses can be welfare-enhancing in cases of dominant platforms, as in the cases of iTunes, Uber and PS Plus, regardless of whether there are many or few sellers. We also note that this result stands in stark contrast with that found in vertical relations settings. In such cases, price-forcing contracts are well-known to damage welfare even if they are profitable for an upstream monopolistic supplier.

7.2. Agency vs. wholesale models of sales. Our model can also shed some light on the private and social desirability of the "agency model of sales" (Johnson, 2014; Gaudin and White, 2014; Gilbert, 2015), which is used, for example, by Apple for selling e-books on its platform. In the

²²Alternatively, the platform could force sellers to commit to the contractually-specified prices before buyers buy the platform.

²³See <http://www.gamasutra.com/view/feature/191966/>, accessed June 26, 2018, for more details.

agency model of sales, the platform acts as a sales agent of sellers, charging a fee for selling their goods through its platform, and sellers still set the price at which they want to sell their goods.²⁴ In terms of our model, the agent (platform) privately offers a two-part tariff (f_i, w_i) to seller i , and this seller chooses price p_i for its good. Therefore, the agency model of sales is equivalent to the two-sided platform studied in Section 4 and leads to similar informational problems.

The alternative to the agency model of sales is the wholesale, merchant, or retail model, used for example by Amazon for selling e-books before 2010. In the wholesale or merchant mode, the platform buys the goods from sellers, and sells them directly to buyers. In our setting, this corresponds to sellers becoming upstream suppliers of the platform, and to the platform provider becoming a downstream retailer that sells both the platform and the sellers' goods to buyers. Because the provider of the platform not only controls directly the platform's price but also the price of platform-based goods, it can include prices for sellers' goods in its contract with buyers. Given that the platform has all the bargaining power when it negotiates with sellers, this model is therefore equivalent to the one in which the platform uses price-forcing contracts studied in the previous section. We have the following result.

Proposition 6 (Agency vs. wholesale models of sales). *Platform profits and consumer surplus are both higher when the platform controls sellers' prices directly (and commits to prices when dealing with buyers) than when it simply controls their royalty fees, so the wholesale model is privately and socially preferred over the agency model of sales.*

It is important to note that in the setting studied in this paper, the only difference between the two models is how they deal with informational problems. If all contracts are observed, both models yield equivalent results. The only reason why the wholesale model outperforms the agency model in our paper is because it provides means for the platform to become more transparent in its dealings with buyers.

The contrast between the agency and wholesale models has recently been made relevant by Amazon's conflicts with book publishers, in particular with French publisher Hachette. Book publishers argued that Amazon was setting too low downstream prices for books, which forced them to accept low upstream prices from Amazon. Apple, in contrast, allowed publishers to set their own prices, and therefore, publishers pushed for Amazon to embrace the agency model, which it did between January and March of 2010.²⁵

Our model gives support to Amazon's claims in its fight with publishers, because it shows that the wholesale model is better apt for solving buyers' informational problems.²⁶ However, in

²⁴The agency model of sales should not be confused with the principal-agent model of information economics.

²⁵For these actions, the US Department of Justice sued Apple and publishers Hachette, Harper Collins, Macmillan, Penguin, and Simon & Schuster, for conspiring to raise the price of e-books in violation of the Sherman Act. See *United States of America v. Apple Inc., et al.*, 12 Civ. 2862 (DLC) for details.

²⁶In its suit against Apple and book publishers, the DOJ presented evidence that book sales fell significantly after Amazon changed its business model, which is consistent with our results.

our setting the platform makes take-it-or-leave-it offers to sellers when bargaining over fees or prices, so sellers' profits are zero in either model (agency or wholesale). To properly understand publishers' incentives, we would need to account for sellers' bargaining power, which we believe is an interesting direction for further research.

Our model also contributes to the recent discussion over the choice between the merchant and two-sided platform business models (Hagiu, 2007; Hagiu and Wright, 2014, 2015, 2016). We show that informational problems can help explain why many platforms operate merchants, when they could instead be run as two-sided platforms, as in the cases of Amazon and Zappos.²⁷ Similarly, our paper helps explaining why firms such as Netflix, Pandora, and Hulu choose to use subscription models with positive membership fees and zero transaction fees. Taking into account that a negative royalty amounts to the platform purchasing the good from sellers, this pricing scheme exactly replicates the price structure of Section 3.

7.3. Integration between the platform and sellers. A natural alternative to contracting with sellers is to acquire them, since this also allows the platform to control the price of platform-based goods. If the contract signed between the platform and the buyers includes not only the access price but also the prices of platform-based goods, all informational frictions again disappear and the situation such as the one in Section 3 arises, so a move towards complete integration would be a Pareto-improvement.

This variant of the model in which the platform is integrated into the production of platform-based goods can be easily used to highlight the importance of commitment by the platform when dealing with buyers, an aspect that applies to all the variants used earlier. When a fully integrated platform cannot commit to the prices charged for sellers' goods, the buyers and the platform face a situation identical to that studied in Section 4.1. Therefore, the lack of commitment lowers platform profits and social welfare, relative to when commitment is feasible. However, integration in the absence of commitment is (privately and socially) preferred over situations in which the platform is not integrated and privately contracts with each seller if and only if $\theta < 0$ (see Figure 6). We have thus shown the following results.

Proposition 7 (Platform integration with sellers). *Under commitment, integration is privately and socially desirable. In the absence of commitment, integration is privately and socially desirable if and only if sellers' goods are complements.*

Proposition 7 contrasts with the findings of previous papers studying vertical relations in one-sided markets, which show that secret contracting is socially preferred over integration. In the case of two-sided markets, in contrast, vertical integration can improve welfare because it helps firms overcome informational problems that cause inefficient hold up problems.

²⁷Interestingly, Amazon operates several business models. For example, Amazon works under the merchant mode for selling products directly to buyers, operates as a two-sided platform when sellers sell goods through Amazon Marketplace, and uses a combination of the merchant and two-sided platform modes for selling music and movies.

7.4. Most favored nation clauses. We conclude this section by analyzing the effect of most favored nation (MFN) clauses.²⁸ For example, the private contract between Spotify (an online music platform) and Sony Music Entertainment (a record company) included MFN clauses.²⁹

Suppose that it is commonly known that the platform includes MFN clauses in their contracts with sellers. Contracts are still private for other sellers and buyers, but sellers are guaranteed not to obtain worse terms than other sellers. We model this situation by assuming that sellers have “symmetry beliefs” (McAfee and Schwartz, 1994). That is, sellers believe that $B(w) = w$ for all w .

It is straightforward to see that with this type of beliefs, results are identical to those of Section 4.1. Therefore, the introduction of MFN clauses makes the platform and buyers worse off when goods are substitutes, but it makes all of them better off when goods are complements. The reason is that MFN clauses solve sellers’ information problems, but not buyers’, which intensifies the hold up problem when sellers’ products are substitutes, and alleviates it when they are complements.³⁰

Proposition 8 (Most Favored Nation clauses). *Relative to when MFN clauses are not used, the introduction of such clauses increases total welfare if sellers’ products are complements and decreases total welfare if they are substitutes.*

So MFN enhance welfare (and platform profits) if and only if products are complements, but they cannot attain the benchmark levels attained under public contracting because the hold up problem faced by buyers does not vanish.

8. FORMATION OF (WEAKLY) PASSIVE BELIEFS BY SELLERS

In this section, we examine the role of belief formation on our conclusions on social welfare and price structures. We assume that sellers form (weakly) passive beliefs (Hart and Tirole, 1990; O’Brien and Shaffer, 1992) rather than wary beliefs when observing an unexpected contract offer.

Relative to the cases in which sellers observe contract offers, sellers anticipate that their interpretation of out-of-equilibrium play may allow the platform to take advantage of them more

²⁸In this section, we study MFN clauses by which the platform commits to charging the same royalties to different sellers. Wang and Wright (2016b,a) study price parity clauses by which sellers commit to charging the same prices across platforms.

²⁹The contract reads: “Company [Spotify] represents, warrants and covenants that, as of the date of execution of this Agreement and throughout the remainder of the Term, it has not entered into any agreement or other understanding with any third party provider of recorded music, videos artwork or other assets used in connection with the Services (each, an “Other Party”) that, includes terms and conditions which, when evaluated as a whole, are more favorable to such Other Party than the terms and conditions, when evaluated as a whole, contained in this Agreement.” Contracts such as this are generally private. However, in the case at hand, the contract was made public by hackers that entered into Sony’s servers. See <http://www.nytimes.com/2015/05/25/business/media/sony-terms-with-spotify-uncovered-in-contract.html>, accessed June 26, 2018, for more details.

³⁰Sony’s contract with Spotify included a clause by which Sony had the right to “request that an independent third party auditor” determine whether Spotify had “entered into a potentially Generally More Favorable Agreement with any Other Party.” Therefore, even though Sony did not have access to Spotify’s contracts with other sellers, it could be sure that those contracts did not include more favorable terms than its own contract. Nevertheless, contracts remained secret for buyers, and thus did not solve their informational problems.

easily. Indeed, an offer involving a high royalty fee when $\theta > 0$ must be associated with a substantially lower fixed fee, reason why the platform is led to lower the royalty fee relative to when seller can observe all contract offers. When $\theta < 0$, the situation is the converse, so the platform is led to increase the royalty fee relative to when seller can observe contract offers. In Appendix C, we show the following result (tildes denote the case in which beliefs are passive).

Proposition 9 (Passive beliefs by sellers). *An equilibrium in which sellers form passive beliefs (uniquely) exists if and only if $\theta \leq 1/2$. When $\theta = 0$, there is no difference in terms of equilibrium outcomes between sellers forming passive beliefs or wary beliefs. When $\theta \leq 1/2$ (with $\theta \neq 0$), it holds that $w^* > \tilde{w} = 0$, $p^* > \tilde{p} > 0$ and $p_0^* < \tilde{p}_0 < 0$, with $\pi_0^* < \tilde{\pi}_0$ and $cs^* < \tilde{cs}$.*

On the one hand, when sellers' beliefs are passive rather than wary, the platform's price structure is somewhat affected, with royalty fees falling down to zero and the markup earned on buyers rising but yet remaining below zero. Platform profits and consumer surplus are still below those attained were contracts public, which is consistent with our analysis in the previous sections.

On the other hand, platform profits and consumer surplus under passive beliefs are larger than under wary beliefs. As is well known from the vertical relations literature, passive beliefs imply a more severe hold up problem in dealing with sellers, which implies that sellers' prices decrease with respect to the case of wary beliefs. In a two-sided platform, this loss of control in dealing with sellers serves as a commitment device for reducing the hold up problem when dealing with buyers, and, as a result, the platform (and buyers) benefit when sellers form passive beliefs rather than wary.

Once again, this result shows how informational problems interact in a two-sided platform, which has a significant effect on the optimal policies that should be implemented from a firm and social perspective. For example, Rey and Vergé (2004) show in the one-sided market case that, when $0 < \theta < 1/2$, an upstream monopolist earns less when downstream firms form passive beliefs than wary beliefs, which is just the opposite of what we find in the presence of two-sidedness.

Our analysis shows that the policy and managerial implications drawn in previous sections remain valid when sellers form passive beliefs.

9. SEQUENTIAL ADOPTION DECISIONS

In previous sections, we assumed that both sides of the market decide whether to join the platform at the same time. Most papers in the two-sided markets literature study this timing (see Rochet and Tirole, 2003; Armstrong, 2006, for example). In fact, one of the main concerns of the two-sided-markets literature is that the mutual dependency of platform-membership decisions may create a chicken-and-egg problem (Caillaud and Jullien, 2003), which is absent when groups of users make their membership decisions sequentially. Other papers assume that the members of one side (typically buyers) make their adoption decisions after observing the adoption decisions

of the members of the other side (typically sellers). In particular, see Hagiu (2006a). In this section, we study a model with this alternative timing and compare it with the model of Section 4.

Both timings are interesting from a practical point of view. Consider, for example, the decision to buy a video game console. When making their choices, gamers consider not only the games that have already been developed for that platform (which would be consistent with the timing of this section), but also the games that will be developed in the future (which is consistent with the timing of Section 4). The timing of Section 4 is likely to be more relevant when a new gaming platform is launched, but both timings are relevant for mature platforms with a large installed base of games.

We focus on the case in which the goods sold by sellers are independent (i.e., $\theta = 0$), and study a two-period model with the following timing.³¹ In the first period, the platform sets p_0 , which becomes public information, and the platform privately offers contract (f_i, w_i) to seller $i \in \{1, 2\}$, which chooses whether or not to accept the contract at the same time the other seller does. In the second period, sellers choose prices for their goods, and then buyers observe all prices and decide whether to join the platform as well as how many units to buy from each seller.

As in the previous sections, we assume that the platform includes p_0 into seller's contracts. Thus, there are no unobservability problems about price p_0 on the seller and the buyer sides. We also continue to assume that sellers have wary beliefs about the royalty fees offered to other sellers.

One would be tempted to speculate that Rey and Vergé's (2004) results extend trivially to this framework, since buyers are fully informed when making their decisions, and sellers' products are, from the point of view of buyer demands, independent. However, there are several differences between Rey and Vergé's (2004) model and this one. First, sellers' goods become complementary: when the price of one product rises, it decreases the number of buyers who join the platform, which has a negative effect on the total demand of the other product. Second, p_0 affects the interaction between sellers, so the platform may use p_0 as an instrument to modulate the effects of private information. Third, beliefs may now depend on p_0 , because: (i) buyer demand depends on p_0 , p_1 and p_2 , so p_0 affects sellers' first-order conditions in the pricing subgames and; (ii) the platform's optimal deviation when it offers a contract to a seller depends on p_0 (which means that p_0 affects the formation of wary beliefs). These effects are absent in Rey and Vergé's (2004) paper.

We start by noting that the public information case of the model with the alternative timing is equivalent to the one studied in Section 3: if all prices are publicly observed, the platform behaves as an integrated firm, induces marginal cost pricing by sellers, and chooses a positive access fee for buyers. Note also that the intermediate case with informed sellers and uninformed buyers

³¹As will soon become clear, the model with sequential adoption decisions is highly complex, which implies that we have to resort to numerical solutions. For concreteness, we focus on the study of the model with $\theta = 0$, and leave the study of the cases with $\theta \neq 0$ for future research.

is equivalent to the public information case of Section 3, since buyers make their consumption decisions after observing p_i (thus, they do not need to form beliefs about royalty fees).

We turn now to the study of the private contracts case. When $\theta = 0$, we know that

$$q_i(p_i) = 1 - p_i \quad (12)$$

and

$$x_0(p_1, p_2, p_0) = \frac{(1 - p_1)^2 + (1 - p_2)^2}{2} - p_0. \quad (13)$$

Suppose that, upon observing w_i and p_0 , seller $i \in \{1, 2\}$ follows pricing strategy $p_i(w_i, p_0)$ and believes that seller j follows pricing strategy $p_j(B(w_i, p_0), p_0)$ in the second period. Because

$$p_i(w_i, p_0) \in \arg \max_{p_i} \{(p_i - w_i)(1 - p_i)x_0(p_i, p_j(B(w_i, p_0), p_0), p_0)\},$$

the first-order condition yields that

$$\begin{aligned} (1 + w_i - 2p_i(w_i, p_0)) \left(\frac{[1 - p_i(w_i, p_0)]^2 + [1 - p_j(B(w_i, p_0), p_0)]^2}{2} - p_0 \right) \\ - [p_i(w_i, p_0) - w_i][1 - p_i(w_i, p_0)]^2 = 0. \end{aligned}$$

Thus, by symmetry, in equilibrium the following functional equation must hold for all w and p_0 :

$$\begin{aligned} (1 + w - 2p(w, p_0)) \left(\frac{[1 - p(w, p_0)]^2 + [1 - p(B(w, p_0), p_0)]^2}{2} - p_0 \right) \\ - [p(w, p_0) - w][1 - p(w, p_0)]^2 = 0. \quad (14) \end{aligned}$$

To obtain the condition that corresponds to the first period, note that, upon observing w_2 and p_0 , seller 2 believes that the platform chooses w_1 to maximize

$$x_0(p_1(w_1, p_0), p_2(w_2, p_0), p_0) [p_0 + w_1 q_1(p_1(w_1, p_0)) + w_2 q_2(p_2(w_2, p_0))] + f_1 + f_2$$

subject to

$$f_1 \leq (p_1(w_1, p_0) - w_1) x_0(p_1(w_1, p_0), p_2(B(w_1, p_0), p_0), p_0) q_1(p_1(w_1, p_0)),$$

so seller 2 believes that the platform chooses w_1 to maximize

$$\begin{aligned} x_0(p_1(w_1, p_0), p_2(w_2, p_0), p_0) [p_0 + w_1 q_1(p_1(w_1, p_0)) + w_2 q_2(p_2(w_2, p_0))] + f_2 + \\ [p_1(w_1, p_0) - w_1] x_0(p_1(w_1, p_0), p_2(B(w_1, p_0), p_0), p_0) q_1(p_1(w_1, p_0)). \quad (15) \end{aligned}$$

Working with the first-order condition we obtain the following differential equation (see Appendix D for details):

$$\begin{aligned}
0 = & - \left[\left(1 - p(B(w, p_0), p_0) \right) \left(p_0 + B(w, p_0) (1 - p(B(w, p_0), p_0)) + w (1 - p(w, p_0)) \right) \right. \\
& + \left(p(B(w, p_0), p_0) - B(w, p_0) \right) (1 - p(B(w, p_0), p_0)) \\
& \times \left(1 - p(B(B(w, p_0)), p_0) \right) \frac{\partial B(B(w, p_0))}{\partial B(w, p_0)} \\
& + \left. \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(w, p_0))^2}{2} - p_0 \right) B(w, p_0) \right] \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)} \\
& + \left(\frac{(1 - p(w, p_0))^2 - (1 - p(B(B(w, p_0)), p_0))^2}{2} \right) (1 - p(B(w, p_0), p_0)). \tag{16}
\end{aligned}$$

Given p_0 , $p(w, p_0)$ and $B(w, p_0)$ solve the system of equations given by (14) and (16). The second-period equilibrium royalty fee, $w^a(p_0)$, solves $w^a = B(w^a, p_0)$. In the first period, the platform chooses p_0 to maximize profits, taking into account that the second-period equilibrium royalty fee is given by $w^a(p_0)$.

A general solution for a system of differential equations is not known. Our attempts to solve the system comprised of equations (14) and (16) were unsuccessful. Thus, we proceeded to obtain a numerical solution. We develop an algorithm using a combination of Euler's method and the fourth-order Runge-Kutta method.³² For a given p_0 , the algorithm yields a numerical approximation of $p(w, p_0)$ and $B(w, p_0)$. Figure 7 shows $p(w, p_0)$ and $B(w, p_0)$ for $p_0 = 0.25$. We apply the algorithm for different values of p_0 and find the one that maximizes the platform's profit. Figure 8 shows first-period profits for the platform for different values of p_0 . The optimal platform price is $p_0^a \approx 0.4785$, the equilibrium royalty fee is $w^a \approx -0.1582$, the equilibrium price for sellers is $p^a \approx 0.1012$, equilibrium platform profits are $\pi^a \approx 0.2172$, and consumer surplus is $cs^a \approx 0.0539$.³³

In comparison, in the public contracts case (Section 3), $p_0^t = 0.5$, $w^t = -1$, $p^t = 0$, $\pi_0^t = 0.25$, and $cs^t = 0.125$; and in the private-contract case with the original timing of Section 4.2, $p_0^* = -0.125$, $w^* = 0$, $p^* = 0.5$, $\pi_0^* \approx 0.1406$, and $cs^* \approx 0.0703$ (when $\theta = 0$, the results of the uninformed buyers case in Section 4.1 coincide with the private contracts case). Our central finding that a strict price control by the platform would benefit both the platform itself and society still holds, even though sellers were assumed to produce goods whose direct demands exhibit neither substitutability nor complementarity.

³²See Butcher (2016) for an introduction to numerical methods for differential equations. Details of the algorithm are available upon request.

³³Given the numerical nature of our solution, these results are approximate. Most importantly, they depend on our choice of: (i) the increment used to calculate numerical derivatives, (ii) the functional form of the approximation error, and (iii) the algorithm's stopping rule. We have conducted sensibility tests which show that the numerical results are robust to changes in these parameters. Results are available upon request.

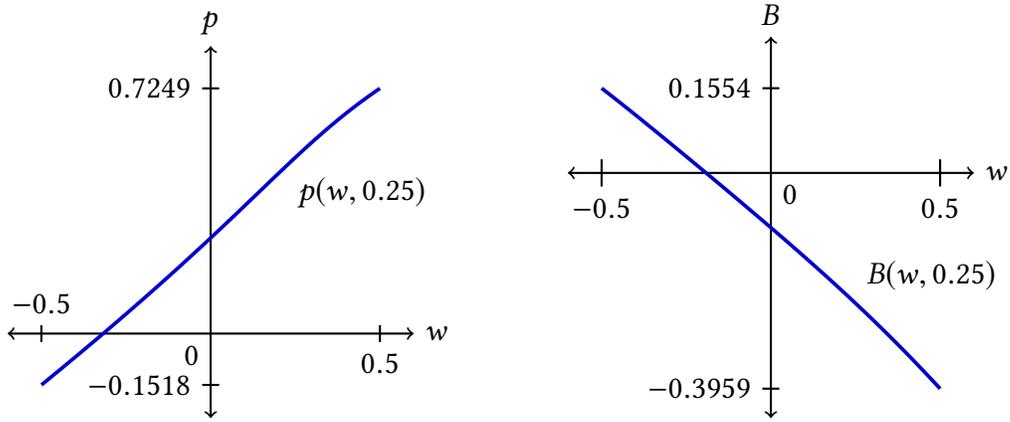


FIGURE 7. Price and belief functions for $p_0 = 0.25$

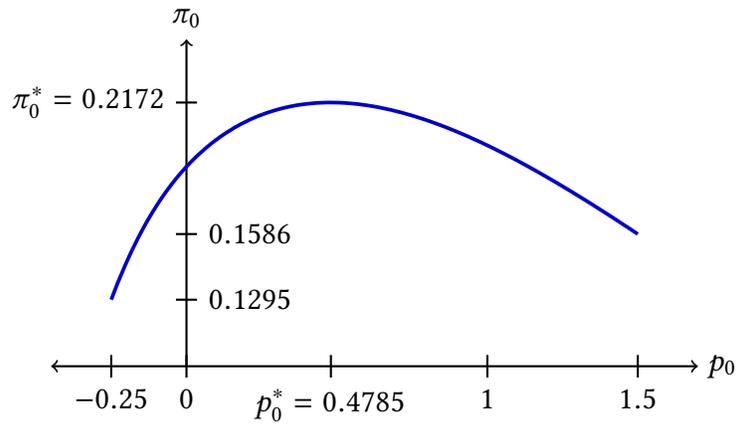


FIGURE 8. Platform profits in the continuation game

To sum up, the analysis of private contracts with sequential adoption shows two main results. First, as in the model with the original timing of Sections 3 and 4, equilibrium profits and consumer surplus are smaller with private contracts than in the public information case, but the price structure with the alternative timing is closer to the price structure of the public contracts case. Second, platform profits are larger and consumer surplus is smaller than in the case of private contracts with the original timing. Therefore, private contracts have a more negative effect on buyers with the alternative timing than with the original timing. These results show that informational problems are important even when buyers make their adoption decisions after sellers. Whereas these results are suggestive, understanding the effects of private contracts when buyers and sellers make adoption decisions at different times requires solving a fully dynamic model, which we leave for further research.

10. CONCLUDING REMARKS

When contracts between the platform and sellers are private rather than public, we have shown that the pricing structure is basically driven by buyers' fear of being taken advantage of when purchasing the platform. Transparency is beneficial because it allows the platform to commit not to trick buyers into purchasing a platform that will have expensive goods sold by sellers. Another way to make this hold up problem disappear is to contract on sellers' prices with both buyers and sellers (as in the case of iTunes). Both the platform and buyers would benefit from the use of such price-forcing contracts. Also, we have shown that integration of the platform with both sellers can allow it to revert to the public contracting outcome and do better without harming buyers, as would happen if the firm becomes a merchant for sellers rather than a sales agent.

Our results show that allowing a dominant platform to strictly control sellers' prices (e.g., in the form of allowing for price-forcing contracts) may make everybody better off because it removes information frictions: everyone benefits from buyers not being so pessimistic about the value delivered by the platform. This insight does not only apply when buyers purchase the platform without observing the costs associated to using it, but rather it is more general. It also holds in cases in which buyers do not observe the quality of the goods sold by sellers before acquiring the platform, or when they do not observe the full variety of goods that will be offered through the platform. This may explain why quality assurance by platforms is common (as is the case for Nintendo), especially when sellers produce experience goods or when buyers need to search for them in order to find their features (price included). We believe that these topics present an interesting direction for further research, and examining whether all these insights carry over to the case of competing platforms is left for future work on the topic.

APPENDIX A. PROOF OF PROPOSITION 3

If $p(w) = \Theta + \Sigma w$ and $B(w) = \Gamma + \Phi w$ for some parameters Θ , Σ , Γ and Φ to be determined, conditions (8) and (11) can be rewritten as

$$(1 - \theta)(1 - \Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) + [2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1)]w_2 = 0$$

and

$$1 - \theta + \theta\Sigma\Gamma - (2 - \theta)\Theta + (1 - 2\Sigma + \theta\Sigma\Phi)w_2 = 0.$$

Since these two conditions should be satisfied for all w_2 , we must have

$$(1 - \theta)(1 - \Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) = 0, \tag{17}$$

$$2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1) = 0, \tag{18}$$

$$1 - \theta + \theta\Sigma\Gamma - (2 - \theta)\Theta = 0, \tag{19}$$

$$1 - 2\Sigma + \theta\Sigma\Phi = 0. \tag{20}$$

Rey and Vergé (2004) have already shown that there exists a unique tuple $(\Theta, \Sigma, \Gamma, \Phi)$ that solves these equations and the required second-order conditions for the platform's maximization program, but we will give closed-form solutions that will prove useful later on.

When $\theta = 0$, it is easy to see that there is a unique solution to equations (17)-(20), given by $\Theta = 1/2, \Sigma = 1/2, \Gamma = 0$ and $\Phi = 0$. From (20), one obtains

$$\Phi = \frac{2\Sigma - 1}{\theta\Sigma},$$

since it can be shown that there can be no solution with $\Sigma = 0$. Plugging this value for Φ in (18) allows us to rewrite it as the following cubic equation:

$$\Sigma^3 - \left(\frac{7 - \theta^2}{2}\right)\Sigma^2 + \frac{5}{2}\Sigma - \frac{1}{2} = 0. \quad (21)$$

Letting $a \equiv -\frac{7-\theta^2}{2}$, $b \equiv \frac{5}{2}$, $c \equiv -\frac{1}{2}$, $K \equiv \frac{3b-a^2}{9}$, and $L \equiv \frac{9ab-27c-2a^3}{54}$, the solutions to the cubic equation are the following:

$$\Sigma_k = 2\sqrt{-K} \cos\left(\frac{1}{3} \arccos\left(\frac{L}{\sqrt{-K^3}}\right) + \frac{2\pi k}{3}\right) - \frac{a}{3} \quad (k = 0, 1, 2).$$

The three roots are real, given that the discriminant $K^3 + L^2$ is negative for all $\theta \in (-1, 1)$. Plotting the three roots for all values of θ , it is easy to see that the only one which is equal to $1/2$ when $\theta = 0$ is Σ_2 . Given that the solution must be continuous in θ , we know that $\Sigma = \Sigma_2$, that is,

$$\Sigma = \frac{7 - \theta^2}{6} - \frac{(19 - 14\theta^2 + \theta^4)^{1/2}}{3} \sin\left(\frac{\pi}{6} - \frac{1}{3} \arccos\left(\frac{(1 - \theta^2)(82 - 20\theta^2 + \theta^4)}{(19 - 14\theta^2 + \theta^4)^{3/2}}\right)\right).$$

From equation (19), we obtain

$$\Gamma = \frac{(2 - \theta)\Theta - (1 - \theta)}{\theta\Sigma},$$

so plugging it into (17) and rearranging yields that

$$\Theta = \frac{(1 - \theta)[(6 + \theta)\Sigma - 2(1 + \Sigma^2)]}{4(3 - \Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

It therefore follows from (17) that

$$\Gamma = \frac{(1 - \theta)(2\Sigma - 1)}{4(3 - \Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

Making it explicit that Θ, Σ, Γ and Φ depend on θ by writing $\Theta_\theta, \Sigma_\theta, \Gamma_\theta$ and Φ_θ , it is easy to check that $0 \leq \Theta_\theta \leq 1, 1/2 \leq \Sigma_\theta \leq 1, 0 \leq \Gamma_\theta \leq 1$ and $-1 \leq \Phi_\theta \leq 1$ for all $\theta \in (-1, 1)$. Note that beliefs must be fulfilled in equilibrium, so $w^* = B(w^*)$ implies that

$$w^* = \frac{\Gamma_\theta}{1 - \Phi_\theta} \geq 0.$$

Also, the platform should find it optimal to choose $p_0 = p_0^*$ and $w_1 = w_2 = w^*$, so $(w^*, w^*, p_0^*) \in \operatorname{argmax}_{w_1, w_2, p_0} \hat{\pi}_0(w_1, w_2, p_0)$, where

$$\begin{aligned} \hat{\pi}_0(w_1, w_2, p_0) = & x_0(p_0) \{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \\ & + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(B(w_1))) \\ & + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \}. \end{aligned}$$

Note that the optimal choices of w_1 and w_2 do not depend on the choice of p_0 , so the platform can maximize with respect to w_1 and w_2 ignoring the value of p_0 ; the analysis above leading to expression (11) shows that private offers are chosen optimally, since second-order conditions are satisfied. To see this, note that (10) and the fact that

$$\frac{dq_1(p_1(w_1), p_2(B(w_1)))}{dw_1} = \frac{1}{1 - \theta^2} \left(\frac{dp_1(w_1)}{dw_1} - 1 \right),$$

imply that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} = \frac{2(\Sigma_\theta^2 - 3\Sigma_\theta + 1)}{1 - \theta^2}$$

and

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} = \frac{2\theta\Sigma_\theta}{1 - \theta^2}.$$

Thus, it follows from the fact that $\Sigma_\theta \geq 1/2$ that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \leq 0.$$

Also, it holds that

$$\begin{aligned} & \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \right)^2 - \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} \right)^2 \\ & = \frac{\Sigma_\theta(\Sigma_\theta^2 - 3\Sigma_\theta + 1)(\Sigma_\theta - 1) - (2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) - \theta^2\Sigma_\theta^2}{\left(\frac{1-\theta^2}{2}\right)^2}, \end{aligned}$$

which is nonnegative because $1/2 \leq \Sigma_\theta \leq 1$ and $(2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) + \theta^2\Sigma_\theta^2 = 0$ by (21). Thus, second-order conditions hold.

As for the optimal choice of p_0 given that seller $i \in \{1, 2\}$ receives an offer equal to (w^*, f^*) , we need that

$$x_0(p_0) + [p_0 + 2p_1(w^*)q_1(p_1(w^*), p_2(w^*))] \frac{dx_0(p_0)}{dp_0} = 0,$$

so

$$p_0^* = \frac{(1 - w^*)^2}{2(1 + \theta)(2 - \theta)^2} - \frac{2(\Theta_\theta + \Sigma_\theta w^*)(1 - \Theta_\theta - \Sigma_\theta w^*)}{2(1 + \theta)},$$

which is negative for all $\theta \in (-1, 1)$. Finally, note that

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \quad (i \in \{1, 2\}),$$

so $0 \leq p_i^* \leq 1$.

It readily follows that $q_i^* = \frac{1-p_i^*}{1+\theta} > 0$, $\pi_0^* = \left(\frac{1-p_i^{*2}}{2(1+\theta)}\right)^2$, and $cs^* = \frac{1}{2} \left(\frac{1-p_i^{*2}}{2(1+\theta)}\right)^2$.

APPENDIX B. DETERMINANTS OF THE NONNEGATIVENESS OF EQUILIBRIUM ROYALTY FEES WHEN CONTRACTS ARE PRIVATE

Taking into account that the platform extracts all the surplus that seller i expects to make when observing royalty fee w_i , it holds that the payoff to the platform if it chooses w_1 , w_2 and p_0 equals

$$\begin{aligned} \widehat{\pi}_0(w_1, w_2, p_0) &= x_0(w^*, p_0) \left\{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \right. \\ &\quad + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(B(w_1))) \\ &\quad \left. + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \right\} \\ &= x_0(w^*, p_0) \left\{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \right. \\ &\quad + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(w_2)) \\ &\quad + [p_1(w_1) - w_1] [q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))] \\ &\quad \left. + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \right\}. \end{aligned}$$

Clearly, maximizing this payoff with respect to w_1 is equivalent to maximizing

$$\begin{aligned} \check{\pi}_0(w_1, w_2) &= [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(w_2)) \\ &\quad + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \\ &\quad + [p_1(w_1) - w_1] [q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))], \end{aligned}$$

so the platform cares about seller 1's actual profit, the actual royalty revenue generated by each seller and the change in seller 1's profit because of the formation of wary beliefs. By the envelope theorem, seller 1's actual profit variation when w_1 changes by a small amount is $-q_1(p_1(w_1), p_2(w_2))$, so $w^* = B(w^*)$ implies that

$$\left. \frac{\partial \check{\pi}_0(w_1, w_2)}{\partial w_1} \right|_{w_1=w_2=w^*} = 0$$

is equivalent to

$$\left\{ [p(w^*) - w^*] \theta \frac{dB(w^*)}{dw} - (1 - \theta) w^* \right\} \frac{dp(w^*)}{dw} = 0.$$

The fact that $p(w^*) > w^*$ then implies that

$$w^* = \frac{\theta}{1 - \theta} \frac{dB(w^*)}{dw} [p(w^*) - w^*]$$

must be nonnegative because $\theta(dB(w^*)/dw) \geq 0$.

As we have shown, the sign of w^* depends on how the second argument of $q_1(p_1(w_1), p_2(B(w_1)))$ varies with w_1 , that is, on whether an increase in w_1 will make seller 1 believe that its sales increase because of the conjectured price change performed by seller 2. Because seller 1 always believes that this is indeed the case, w^* is always nonnegative.

APPENDIX C. PROOF OF PROPOSITION 9

Suppose that, in a symmetric PBE sustained by passive beliefs, the platform must charge \tilde{p}_0 and must offer contract (\tilde{f}, \tilde{w}) to each seller. Consider, however, what happens if one of them denoted $i \in \{1, 2\}$ observes p_0 and (f_i, w_i) , where p_0 , f_i and w_i need not coincide with \tilde{p}_0 , \tilde{f} or \tilde{w} , and let us examine what happens if such a seller forms passive beliefs. The seller anticipates that the other seller must charge

$$\tilde{p} = \frac{1 - \theta + \tilde{w}}{2 - \theta},$$

so seller i anticipates charging

$$\tilde{p}_i(w_i) = \frac{1 - \theta + w_i + \theta\tilde{p}}{2}.$$

Seller i also anticipates that the buyers' demand for the platform must equal

$$x_0(p_0, \tilde{w}) = \frac{(1 - \tilde{w})^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

Regardless of what it observes, seller i believes that seller $j \neq i$ is offered \tilde{w} and hence expects i to observe equilibrium play and thus charge \tilde{p} . As a result, the profit of the platform if it charges p_0 to buyers and it offers (f_i, w_i) to seller $i \in \{1, 2\}$ is

$$\tilde{\Pi}(w_1, w_2, p_0) = x_0(p_0, \tilde{w}) \left[p_0 + w_1 \left(\frac{1 - \theta - \tilde{p}_1(w_1) + \theta\tilde{p}_2(w_2)}{1 - \theta^2} \right) + w_2 \left(\frac{1 - \theta - \tilde{p}_2(w_2) + \theta\tilde{p}_1(w_1)}{1 - \theta^2} \right) \right] + f_1 + f_2,$$

where

$$f_i = x_0(p_0, \tilde{w})(\tilde{p}_i(w_i) - c - w_i) \left(\frac{1 - \theta - \tilde{p}_i(w_i) + \theta\tilde{p}}{1 - \theta^2} \right), \quad i = 1, 2.$$

Equivalently,

$$\tilde{\Pi}(w_1, w_2, p_0) = x_0(p_0, \tilde{w}) \left[p_0 + \sum_{i=1}^2 \sum_{j=1; j \neq i}^2 w_i \left(\frac{1 - \theta - \tilde{p}_i(w_i) + \theta\tilde{p}_j(w_j)}{1 - \theta^2} \right) + \sum_{i=1}^2 (\tilde{p}_i(w_i) - c - w_i) \left(\frac{1 - \theta - \tilde{p}_i(w_i) + \theta\tilde{p}}{1 - \theta^2} \right) \right].$$

It is clearly optimal for the platform to deviate in the same way with respect to the royalty fees, so letting $w_i = w$ implies that its payoff function can be written as follows:

$$\tilde{\Pi}(w, p_0) = \left(\frac{(1 - c - \tilde{w})^2}{(1 + \theta)(2 - \theta)^2} - p_0 \right) \left[p_0 + w \left(\frac{1 - c + \theta - w - \theta\tilde{p}}{1 + \theta} \right) + \frac{(1 - c - \theta - w + \theta\tilde{p})^2}{2(1 - \theta^2)} \right].$$

The first-order condition with respect to w yields that

$$\left(\frac{(1-c-\tilde{w})^2}{(1+\theta)(2-\theta)^2} - p_0 \right) \left[\frac{c\theta + \theta(1-\theta) + (2\theta-1)\tilde{w} - \theta(2-\theta)\tilde{p}}{1-\theta^2} \right] = 0.$$

Because the solution to such first-order condition must be $w = \tilde{w}$, one obtains that $\tilde{w} = 0$, so the platform does not distort pricing by sellers (this result is in line with Rey and Vergé, 2004). Taking this result into account, the first-order condition with respect to p_0 is

$$\frac{(1-c)^2}{(1+\theta)(2-\theta)^2} - 2p_0 - \frac{[1-c-\theta + \theta(\frac{1+c-\theta}{2-\theta})]^2}{2(1-\theta^2)} = 0.$$

Therefore,

$$\tilde{p}_0 = -\frac{(1-2\theta)(1-c)^2}{2(1+\theta)(2-\theta)^2},$$

so the platform is sold at a loss to buyers, as when sellers form wary beliefs.

It simply remains to verify that \tilde{w} and \tilde{p}_0 globally maximize $\tilde{\Pi}(w, p_0)$. Since

$$\frac{\partial \tilde{\Pi}(w, p_0)}{\partial w} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} = \frac{\partial \tilde{\Pi}(w, p_0)}{\partial p_0} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} = 0,$$

we simply need to check that

$$\begin{aligned} 0 &\leq \left(\frac{\partial^2 \tilde{\Pi}(w, p_0)}{\partial p_0^2} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} \right) \left(\frac{\partial^2 \tilde{\Pi}(w, p_0)}{\partial w^2} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} \right) - \left(\frac{\partial^2 \tilde{\Pi}(w, p_0)}{\partial w \partial p_0} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} \right)^2 \\ &= (-2) \frac{(1-2\theta)[\tilde{p}_0(1+\theta)(2-\theta)^2 - (1-c)^2]}{(1-\theta)(1+\theta)^2(2-\theta)^2}, \end{aligned}$$

where the last equality makes use of the fact that

$$\frac{\partial^2 \tilde{\Pi}(w, p_0)}{\partial w \partial p_0} \Big|_{w=\tilde{w}, p_0=\tilde{p}_0} = 0.$$

As a result of straightforward algebra, \tilde{w} and \tilde{p}_0 globally maximize $\tilde{\Pi}(w, p_0)$ if and only if $\theta \leq 1/2$. This is exactly the necessary and sufficient existence condition obtained by Rey and Vergé (2004) for the case of passive beliefs in one-sided markets.

Since $\tilde{w} = 0$, it holds that $w^* \geq \tilde{w}$, with equality if and only if $\theta = 0$. Figure 9 shows that the price charged by sellers when they form passive beliefs is lower than that when beliefs are wary (the dashed curve corresponds to the case of wary beliefs).

Figure 10 shows that the platform's price under passive beliefs is greater than that under wary beliefs (the dashed curve corresponds to the case of wary beliefs).

Finally, profits under passive beliefs equal

$$\tilde{\pi}_0 = \Pi(\tilde{w}, \tilde{p}_0) = \frac{(3-2\theta)^2}{4(1+\theta)^2(2-\theta)^4}.$$

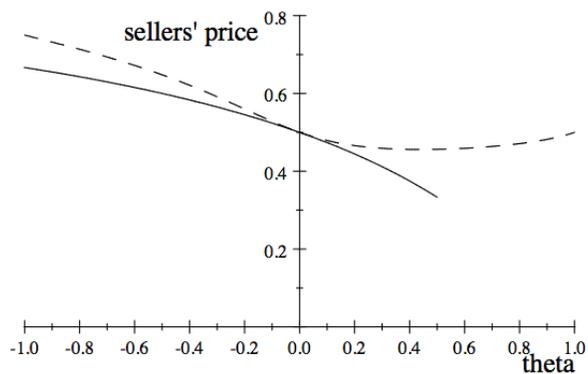


FIGURE 9. Comparison of seller prices

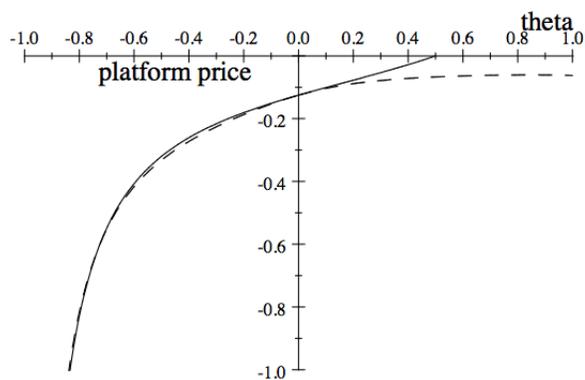


FIGURE 10. Comparison of platform's access price

These profits exceed those under wary beliefs, as illustrated by Figure 11 (the dashed curve corresponds to the case of wary beliefs).

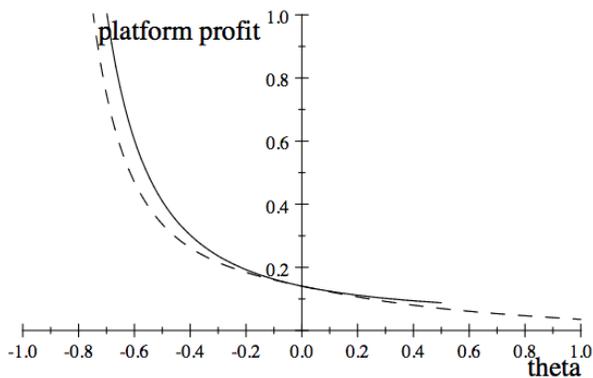


FIGURE 11. Comparison of platform profit

Consumer surplus is half of the platform's profit when beliefs are wary, and the same applies to passive beliefs, since

$$\begin{aligned}\tilde{cs} &= \int_0^{x_0} \left(\frac{(1-c-\tilde{w})^2}{(1+\theta)(2-\theta)^2} - \tilde{p}_0 - x \right) dx \\ &= \frac{(3-2\theta)^2}{8(1+\theta)^2(2-\theta)^4}.\end{aligned}$$

APPENDIX D. PROCEDURE FOR OBTAINING EQUATION (16)

Introducing (12) and (13) into (15), we obtain

$$\begin{aligned}&\left(\frac{(1-p_1(w_1, p_0))^2 + (1-p_2(w_2, p_0))^2}{2} - p_0 \right) \left(p_0 + w_1(1-p_1(w_1, p_0)) + w_2(1-p_2(w_2, p_0)) \right) + f_2 + \\ &\left(\frac{(1-p_1(w_1, p_0))^2 + (1-p_2(B(w_1, p_0), p_0))^2}{2} - p_0 \right) \left(p_1(w_1, p_0) - w_1 \right) \left(1 - p_1(w_1, p_0) \right).\end{aligned}$$

The first-order condition with respect to w_1 is

$$\begin{aligned}0 &= -\left(1 - p_1(w_1, p_0) \right) \frac{\partial p_1(w_1, p_0)}{\partial w_1} \left(p_0 + w_1(1-p_1(w_1, p_0)) + w_2(1-p_2(w_2, p_0)) \right) \\ &+ \left(\frac{(1-p_1(w_1, p_0))^2 + (1-p_2(w_2, p_0))^2}{2} - p_0 \right) \left(1 - p_1(w_1, p_0) - w_1 \frac{\partial p_1(w_1, p_0)}{\partial w_1} \right) \\ &- \left(\left(1 - p_1(w_1, p_0) \right) \frac{\partial p_1(w_1, p_0)}{\partial w_1} + \left(1 - p_2(B(w_1, p_0), p_0) \right) \frac{\partial p_2(B(w_1, p_0), p_0)}{\partial B(w_1, p_0)} \frac{\partial B(w_1, p_0)}{\partial w_1} \right) \\ &\times \left(p_1(w_1, p_0) - w_1 \right) \left(1 - p_1(w_1, p_0) \right) \\ &+ \left(\frac{(1-p_1(w_1, p_0))^2 + (1-p_2(B(w_1, p_0), p_0))^2}{2} - p_0 \right) \\ &\times \left(\left(\frac{\partial p_1(w_1, p_0)}{\partial w_1} - 1 \right) \left(1 - p_1(w_1, p_0) \right) - \left(p_1(w_1, p_0) - w_1 \right) \frac{\partial p_1(w_1, p_0)}{\partial w_1} \right).\end{aligned}$$

Replacing w_2 by w , and w_1 by $B(w_2, p_0)$, and noting that $p_1(\cdot)$ and $p_2(\cdot)$ should be equal because of symmetry, we obtain

$$\begin{aligned}
0 &= -\left(1 - p(B(w, p_0), p_0)\right) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)} \\
&\quad \times \left(p_0 + B(w, p_0) (1 - p(B(w, p_0), p_0)) + w (1 - p(w, p_0))\right) \\
&\quad + \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(w, p_0))^2}{2} - p_0\right) \\
&\quad \times \left(1 - p(B(w, p_0), p_0) - B(w, p_0) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)}\right) \\
&\quad - \left[\left(1 - p(B(w, p_0), p_0)\right) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)}\right. \\
&\quad \left.+ \left(1 - p(B(B(w, p_0)), p_0)\right) \frac{\partial p(B(B(w, p_0)), p_0)}{\partial B(B(w, p_0))} \frac{\partial B(B(w, p_0))}{\partial B(w, p_0)}\right] \\
&\quad \times \left(p(B(w, p_0), p_0) - B(w, p_0)\right) (1 - p(B(w, p_0), p_0)) \\
&\quad + \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(B(B(w, p_0)), p_0))^2}{2} - p_0\right) \\
&\quad \times \left(\left(\frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)} - 1\right) (1 - p(B(w, p_0), p_0))\right. \\
&\quad \left.- \left(p(B(w, p_0), p_0) - B(w, p_0)\right) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)}\right).
\end{aligned}$$

Together with (14), this differential equation, that should hold for all w and p_0 , gives $p(w, p_0)$ and $B(w, p_0)$.

Note that (14) implies that

$$\begin{aligned}
&\left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(B(B(w, p_0)), p_0))^2}{2} - p_0\right) \\
&\times \left(\left(1 - p(B(w, p_0), p_0)\right) - \left(p(B(w, p_0), p_0) - B(w, p_0)\right)\right) \\
&= \left(1 - p(B(w, p_0), p_0)\right)^2 (p(B(w, p_0), p_0) - B(w, p_0)).
\end{aligned}$$

Thus, the above expression simplifies to

$$\begin{aligned}
0 &= -\left(1 - p(B(w, p_0), p_0)\right) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)} \\
&\quad \times \left(p_0 + B(w, p_0)(1 - p(B(w, p_0), p_0)) + w(1 - p(w, p_0))\right) \\
&\quad + \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(w, p_0))^2}{2} - p_0\right) \\
&\quad \times \left(1 - p(B(w, p_0), p_0) - B(w, p_0) \frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)}\right) \\
&\quad - \left(\left(1 - p(B(B(w, p_0)), p_0)\right) \frac{\partial p(B(B(w, p_0)), p_0)}{\partial B(B(w, p_0))} \frac{\partial B(B(w, p_0))}{\partial B(w, p_0)}\right) \\
&\quad \times \left(p(B(w, p_0), p_0) - B(w, p_0)\right) (1 - p(B(w, p_0), p_0)) \\
&\quad - \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(B(B(w, p_0)), p_0))^2}{2} - p_0\right) \left(1 - p(B(w, p_0), p_0)\right),
\end{aligned}$$

which is equivalent to equation (16).

REFERENCES

- ARMSTRONG, M. (2006): "Competition in two-sided markets," *RAND Journal of Economics*, 37(3), 668–691.
- BUTCHER, J. C. (2016): *Numerical Methods for Ordinary Differential Equations*. Wiley, New York, NY.
- CAILLAUD, B., AND B. JULLIEN (2001): "Competing cybermediaries," *European Economic Review*, 45(4), 797–808.
- (2003): "Chicken & egg: Competition among intermediation service providers," *RAND Journal of Economics*, 34(2), 309–328.
- CASADESUS-MASANELL, R., AND G. LLANES (2015): "Investment Incentives in Open-Source and Proprietary Two-Sided Platforms," *Journal of Economics & Management Strategy*, 24(2), 306–324.
- EVANS, D. S., AND R. SCHMALENSEE (2015): "The Antitrust Analysis of Multi-Sided Platform Businesses," in *Oxford Handbook on International Antitrust Economics*, ed. by R. Blair, and D. Sokol, vol. 1, chap. 18, pp. 404–450. Oxford University Press, Oxford, UK.
- GAUDIN, G., AND A. WHITE (2014): "On the antitrust economics of the electronic books industry," Working paper.
- GILBERT, R. J. (2015): "E-books: A Tale of Digital Disruption," *The Journal of Economic Perspectives*, 29(3).
- HAGIU, A. (2006a): "Pricing and commitment by two-sided platforms," *The RAND Journal of Economics*, 37(3), 720–737.
- (2006b): "Proprietary vs. open two-sided platforms and social efficiency," Working paper 09-113, Harvard Business School, Boston.

- (2007): “Merchant or two-sided platform?,” *Review of Network Economics*, 6(2), 115–133.
- HAGIU, A., AND H. HAŁABURDA (2014): “Information and two-sided platform profits,” *International Journal of Industrial Organization*, 34, 25–35.
- HAGIU, A., AND J. WRIGHT (2014): “Marketplace or reseller?,” *Management Science*, 61(1), 184–203.
- (2015): “Multi-sided platforms,” *International Journal of Industrial Organization*, 43, 162–174.
- (2016): “Controlling Versus Enabling,” Working paper, Harvard Business School, Boston, MA.
- HALABURDA, H., M. J. PISKORSKI, AND P. YILDIRIM (forthcoming): “Competing by restricting choice: The case of search platforms,” *Management Science*.
- HART, O., AND J. TIROLE (1990): “Vertical Integration and Market Foreclosure,” *Brookings Papers on Economic Activity: Microeconomics*, pp. 205–286.
- JOHNSON, J. P. (2014): “The agency model and MFN clauses,” Working paper, Cornell University, Ithaca, NY.
- MC AFEE, R. P., AND M. SCHWARTZ (1994): “Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity,” *American Economic Review*, 84(1), 210–230.
- O’ BRIEN, D. P., AND G. SHAFFER (1992): “Vertical control with bilateral contracts,” *RAND Journal of Economics*, 23(3), 299–308.
- REY, P., AND T. VERGÉ (2004): “Bilateral control with vertical contracts,” *RAND Journal of Economics*, 35(4), 728–746.
- ROCHET, J.-C., AND J. TIROLE (2003): “Platform competition in two-sided markets,” *Journal of the European Economic Association*, 1(4), 990–1029.
- (2006): “Two-sided markets: A progress report,” *RAND Journal of Economics*, 37(3), 645–667.
- VIVES, X. (2001): *Oligopoly pricing: old ideas and new tools*. MIT press, Cambridge, MA.
- WANG, C., AND J. WRIGHT (2016a): “Platform Investment and Price Parity Clauses,” Working paper.
- (2016b): “Search platforms: Showrooming and price parity clauses,” Working paper.