PRIVATE CONTRACTS IN TWO-SIDED MARKETS

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Abstract. We study a two-sided market in which a platform that connects consumers and sellers signs private contracts with sellers. We compare this situation with a two-sided market with public contracts. We find that the platform provider sets positive (negative) royalties to sellers and earns a negative (positive) markup on consumers when contracts are private (public). Thus, private contracting has a significant effect on the price structure. We also find that private contracting leads to lower platform profits, consumer surplus, and social welfare. We study the welfare effects of most favored nation clauses, price-forcing contracts, and integration between platform and sellers; and relate our results with the agency model of sales. Our results indicate that enhancing the market power of a dominant platform over sellers may increase welfare because it acts as a commitment device for inducing lower seller prices, mitigating the hold-up problem borne by consumers when they cannot observe sellers' contracts. Thus, policy prescriptions are completely reversed in the case of a two-sided market, in comparison with the one-sided setting studied by the vertical relations literature.

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1. Introduction

Private contracts are common in two-sided markets. For example, Amazon signs private contracts with publishers, Netflix with movie studios, Sony and Nvidia with videogame developers, Spotify with record companies, HMOs with healthcare providers, Google with phone manufacturers, Apple with cellphone carriers, and Intel and Microsoft with computer manufacturers. In this paper, we show that private contracting has a critical impact on the platform's price structure, industry profitability, and social welfare. Besides helping to explain many commonly observed features of two-sided markets, we also show that many policies that are found to be anticompetitive in one-sided markets (e.g., vertical integration and resale price maintenance) actually improve welfare in two-sided markets with private contracts.

We study a two-period model of a platform that connects buyers and sellers of platform-based products. Such products may be substitutes or complements. In the first period, the platform provider chooses the membership or access fees to be paid by buyers and sellers, and sets the royalty fees that sellers have to pay for each unit of the good they sell to consumers; then sellers decide whether to accept the two-part-tariff contract offered by the platform provider, and consumers decide whether to join the platform. In the second period, sellers post prices and consumers who have purchased access to the platform choose how much to buy from each seller.[1]

Our aim is to compare different information structures and shed light on how each of them affects equilibrium royalty fees, access prices, platform profits, and welfare. In particular, we perform a comparison between: (i) the *public contracts* case, in which the platform provider's pricing scheme is publicly observable –the standard assumption in the two-sided markets literature (see Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006; Hagiu, 2006; Weyl, 2010, for example)– and; (ii) the *private contracts* case, in which the platform's offer to each seller is observed only by that seller –one of the critical foundations in the modern approach to vertical relations in the literature (see Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004, for example).

When the contract offered by the platform provider to a seller is private, equilibrium behavior depends on how sellers and buyers form beliefs about other players' private information when they observe out-of-equilibrium play. In line with the literature, we assume that consumers form "passive beliefs" (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Hagiu and Hałaburda, 2014) and sellers form "wary beliefs" (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when observing unexpected behavior by the platform. Thus, a consumer who observes a price for the platform that differs from the one expected in equilibrium believes that sellers' (pricing) behavior is unaffected, and a seller observing an unexpected two-part

[1] We also study a model in which consumers decide to join the platform after observing seller prices, and discuss how results may change with this alternative timing.

tariff believes that the platform provider is acting opportunistically when pursuing such a deviation from expected play. In particular, the seller (rightly) conjectures that the platform provider has deviated in a profit-maximizing manner with the other seller.[2]

We find that the conclusions drawn from a model of a two-sided market with private contracts stand in stark contrast with those of a model with public contracts. When contracts are public, equilibrium royalty fees are *negative* and the platform provider's markup on consumers is *positive*. When contracts are private, in contrast, royalty fees are *positive* and the platform provider's markup on consumers is *negative*. Thus, the price structure is completely overturned when contracts are private instead of public. Our results explain the price patterns observed in many industries in which contracts are private (for example, video game consoles and ebook readers are generally sold at a price below marginal cost and games and ebooks are sold at a positive markup).

We also find that private contracting results in lower profit for the platform provider, as well as lower consumer and social surplus. This result contrasts with those of papers studying private contracts in one-sided markets (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004), in which private contracting lowers industry profits but increases consumer surplus and welfare.

To understand our results, start by considering the public contracts case. On the one hand, the platform captures sellers' profits through the fixed fee and (indirectly) controls their pricing through royalty fees. Sellers' prices have a dual impact on consumer demands: they affect per-capita demand for sellers' products and consumers' demand for platform access. On the other hand, the platform has direct control on the consumers' access prices. Such prices affect the demand for platform access, but they do not distort per-capita demands for sellers' products, so they are a more efficient instrument for extracting consumers' rents. As a consequence, the platform provider chooses royalties to induce marginal cost pricing by sellers and charges positive access fees to buyers.

Consider now an intermediate case in which sellers observe all contractual offers, but sellers' contracts are unobserved by consumers. Consumers anticipate that the platform will behave opportunistically, choosing the unobservable royalty fees to induce collusive pricing by sellers once the consumers have paid the platform's access fee. This hold-up problem, correctly anticipated by consumers, lowers their demand for platform access. In comparison with the public contracts case, the platform provider sets lower consumer access prices for two reasons: to compensate the decrease in consumer demand for platform access, and because seller revenue per consumer increases (given that seller prices are set at the collusive level, each consumer who joins the platform becomes more valuable in terms of

[2]We also show that our insights persist when sellers form passive beliefs rather than wary beliefs. Wary beliefs mitigate the opportunistic behavior of the platform provider relative to having sellers form passive beliefs. It is well-known from Rey and Vergé (2004) that there might exist no equilibria if sellers form passive beliefs in a setting like the one we shall consider.

the revenues she generates when consuming sellers' products). Using a simple revealed preference argument, is is straightforward to show that the effect of the lower demand dominates the effect of higher prices, and platform's profits decrease as a result. Consumer surplus also decreases, since fewer consumers join the platform and the surplus each one of them obtains is lower.

Finally, consider the private contracts case (in which sellers' contracts are secret to consumers and to other sellers), and assume for the time being that sellers' goods are substitutes. In this case, one may be tempted to extrapolate Rey and Vergé's (2004) finding that the platform provider must be worse off (relative to the intermediate case mentioned above), for it loses part of its market power vis-à-vis sellers. Such an extrapolation would be incorrect because it would miss the feedback loops that arise in a two-sided market.

In particular, we find that in a two-sided market, decreasing the market power on one side may enhance market power on the other side.[3] Sellers fear that the platform will behave opportunistically, offering lower royalties to other sellers when they accept their contract. Thus, the royalties that sellers are willing to accept are lower, which implies that royalties and sellers' prices decrease relative to the intermediate case. The fear of opportunism of sellers effectively makes the platform provider lose some control about sellers' prices, but partly offsets the fear that consumers have that the platform provider will behave opportunistically with them, so the foreseen decrease in seller prices encourages consumers to join the platform. Therefore, the lack of commitment when setting sellers' royalties acts as a *commitment device* for inducing lower seller prices, and mitigates the hold-up problem borne by consumers when they cannot observe sellers' contracts. As a result, the platform provider can charge higher access prices to consumers and still increase the number of consumers joining the platform. These effects dominate the smaller profit per consumer that can be extracted from sellers, so platform profits increase relative to the intermediate case in which only buyers are uninformed.

In contrast with the substitutes case, when sellers' products are complementary, the platform earns less from sellers (for a given number of consumers), but also attracts fewer consumers, relative to the intermediate case.[4] The loss of market power by the platform provider makes it less capable of internalizing the double marginalization problem faced by sellers (Cournot, 1838), so consumers expect sellers to charge higher prices. The platform becomes less valuable to buyers, and the hold-up problem becomes more severe. Even though the platform provider charges lower prices to attract consumers, platform sales decrease and the platform provider is harmed by the lower usage of the platform by consumers and the smaller profit appropriated from sellers.

[3]We define market power as a firm's ability to charge prices above marginal cost.
[4]To the best of our knowledge, the complements case has not been analyzed by the vertical relations literature dealing with secret contracts.

Comparing now the public contracts and private contracts cases, it holds when contracts are private rather than public that consumers fear being taken advantage of by the platform because they cannot observe the actual royalties that the platform will receive from sellers. When sellers' goods are substitutes, the hold-up problem that consumers face is *mitigated* by the loss of market power that the platform provider bears when it secretly contracts with each seller. Our contribution in this case is to show that the consumers' initial concern is not mitigated enough by this loss in control. The platform provider's profits are therefore smaller when contracts are private rather than public. Consumer surplus and social welfare decrease as well. When sellers' offer complements instead of substitutes, our contribution is to show that the consumers' concern about the platform provider's opportunistic behavior is *accentuated* because it has less control over the double marginalization problem faced by the sellers. Relative to public contracting, private contracts again result in higher royalties, higher prices charged by sellers, lower prices for the platform, lower profitability for the platform provider, and lower consumer and social welfare.

We also study the welfare effects of most favored nation (MFN) contractual clauses (used, for example, in the secret contract between Spotify and Sony recently disclosed by North Korean hackers), price-forcing contracts (i.e., contracts that allow the platform to contract on sellers' prices, as in the case of PlayStation Plus), and integration between the platform and sellers. We find that MFN clauses increase welfare when seller's products are complements, and reduce welfare when seller's products are substitutes, whereas price-forcing contracts increase welfare in both cases. The difference is that MFN clauses solve the commitment problem with sellers, but not with consumers, whereas *price-forcing contracts solve the commitment problem with sellers and consumers*. Integration with sellers can also help avoiding the welfare losses from private contracting when the integrated platform provider can commit to the prices charged for platform-based goods or if these goods are complements in the absence of such price commitment.

Overall, our findings suggest that enhancing the platform's market power may be beneficial because it prevents consumers from facing a hold-up problem that would harm adoption and overall platform profitability. For instance, contracts in which sellers' prices are set by the platform may be good for the platform provider and for consumers as well. Such price-forcing contracts are so beneficial because they fully alleviate consumers' fears of being held up after joining to the platform. This result explains the pervasive use of price-forcing contracts in two-sided markets (as in the cases of iTunes, Uber, and PS Plus, for example).

Even if platform profitability and welfare would increase if informational frictions disappeared, at least three aspects are worth keeping in mind. First, contract privacy is often an environmental constraint for sellers and platforms (i.e., any contract publicly shown can always be secretly renegotiated, so contracts would be *de facto* private). Second, our results suggest that policies that attempt to remove informational frictions may actually be worse for the platform and consumers if they improve sellers' information but fail to affect consumers' information. Third, and this is precisely one of the main points of our paper, there are other ways in which a platform provider can get around the problems arising from informational frictions (e.g., contracting on sellers' prices). Although these practices would be often treated as anticompetitive in a one-sided market, in the current setting they are beneficial not only for the platform but also for society at large.

Our paper contributes to the literature on two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). To the best of our knowledge, the entire literature assumes that contracts are publicly observable to all parties. The only exception in which one of the two sides does not observe the price charged to the other side is the paper by Hagiu and Halaburda (2014), which examines how price transparency affects market outcomes. Our result that contractual transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers is different from Hagiu and Halaburda's (2014) insight because buyers and sellers do not interact in their setting. In fact, if sellers' prices were contractible in our setting, then the platform would commit not to hold up consumers and consumers would benefit from it (think of iTunes, for example). In contrast with the two-sided markets literature, we also allow sellers to enjoy market power, so the platform provider shapes their competitive interaction through its choice of royalty fees.

Our paper also builds on the literature on vertical relations regulated by secret contracts, with important contributions by Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and especially Rey and Vergé (2004). Using their terminology, the upstream supplier in our setting has another type of customer with whom downstream firms interact, and such interaction is shaped by the upstream supplier's decisions. This two-sidedness of the problem implies that there are cross-group network effects, so the issues and results are very different from this literature. In fact, policy conclusions are completely overturned when studying a two-sided market instead of a one-sided market.

2. The model

We study a two-sided market composed of a platform provider, $n \geq 2$ sellers, and a continuum of consumers. The platform provider produces a platform good (such as a video console) at a normalized marginal cost of zero. Sellers sell platform-specific products (such as video games) to consumers (e.g., gamers) who buy the platform good. Sellers produce at zero marginal cost, again a normalization.[5]

[5]If sellers had a constant marginal cost $c \in [0, 1)$ of production, the normalization would be as follows: sellers' prices should be interpreted as markups, and these (equilibrium) prices, royalty fees and sellers' sales should be multiplied by the scaling factor $(1 - c)$; in turn, the number of consumers buying the platform should be multiplied by the scaling factor $(1 - c)^2$, whereas the surplus attained by the platform and consumers should be multiplied by the scaling factor $(1 - c)^4$.

Consumers are uniformly spread on the positive real line (with unit density), and the platform provider is located at the left end. The utility of a consumer located at distance $x \in [0, \infty)$ from the platform provider if she purchases the platform good at price p_0, and $q_i \geq 0$ units of the product of seller $i \in \{1, ..., n\}$ at price p_i per unit is (see Vives, 2001, for example)

$$U_x(p_0, p_1, q_1, \ldots, p_n, q_n) = u(p_1, q_1, \ldots, p_n, q_n) - x - p_0,$$

where

$$u(p_1, q_1, \ldots, p_n, q_n) = \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + \theta \sum_{i=1}^{n} \sum_{j=1; j \neq i}^{n} q_i q_j\right) - \sum_{i=1}^{n} p_i q_i.$$

Parameter $\theta \in (-1, 1)$ captures the degree of complementarity/substitution between sellers' goods. If $\theta < 0$, goods are complements, with their degree of complementarity decreasing with θ. If $\theta = 0$, goods are independent. If $\theta > 0$, goods are substitutes, with their degree of substitutability increasing with θ.

We consider the following two-period model. In the first period, the platform provider offers a contract to each seller, and commits to a price p_0 for consumers; then sellers decide whether to accept the contracts, and consumers observe both p_0 and how many sellers have accepted the contract before having to decide whether to buy the platform good. In the second period, sellers set prices for their products and consumers decide how many products to buy from each seller. Our timing reflects the fact that consumers use the platform for many periods, during which platform-specific products are continuously being launched. For instance, buyers of a video console often buy it without observing the prices charged for the games they will consume during the lifetime of the console.

Unless otherwise stated, a contract between seller $i \in \{1, ..., n\}$ and the platform provider consists of a fixed fee f_i and a per-unit royalty fee w_i.[6] If seller i accepts the contract and then sells Q_i units to consumers, its total payment to the platform provider is $f_i + w_i\, Q_i$. We will take n as given, and, for the most part of the paper, we will let $n = 2$. We discuss what happens as n grows large in Section 5.

In Section 3, we study a two-sided market with public contracts, so consumers and sellers observe all contracts before making their decisions. In Section 4, we study a two-sided market with private contracts. We first examine an intermediate situation in which consumers do not observe the contracts offered to sellers, but sellers observe all contracts. We then examine the private contracts case, in which consumers do not observe the contracts offered to sellers, and each seller only observes the contract it is offered. We assume throughout that p_0 is contractible and is written in the contract offered to any seller.[7]

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[6]Therefore, the price at which seller i should sell its goods to consumers is not included in the contract. We will examine in Section 6 what would happen with such price-forcing contracts.

[7]In most occasions, p_0 can be contracted upon. Even if p_0 is not contractible, reputational concerns may prevent the platform provider from behaving opportunistically with sellers. That p_0 is known by sellers when

Even though we assume that the number of sellers is fixed, the market is two-sided because the number of consumers who join the platform affects the profitability of sellers. Thus, there are indirect network effects between buyers and sellers. In Section 3, we also show that the platform's price structure has a non-trivial effect on membership decisions and the level of transactions.

In Section 3, we seek for symmetric subgame perfect equilibria (SPE). In Section 4, we seek for symmetric perfect Bayesian equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed (Section 7 shows that results are robust to alternative ways of forming out-of-equilibrium-path beliefs).

3. Public contracts in a two-sided market

We start by studying the second period. After observing p_i and p_i ($i, j \in \{1, 2\}; i \neq j$), consumers who have purchased the platform good decide how many units of sellers' products to consume. Looking at interior solutions of a consumer's utility maximization problem yields the following *per-capita* demand for the product of seller i:

$$q_i(p_i, p_j) = \frac{1 - \theta - p_i + \theta p_j}{1 - \theta^2}. \tag{1}$$

Per-capita consumption does not depend on the distance between the consumer and the platform. Thus, the overall demand for seller i's product is $Q_i(p_i, p_j) = x_0\, q_i(p_i, p_j)$, where x_0 is the number of consumers who choose to buy the platform good in the first period. Seller $i \in \{1, 2\}$ solves the following problem given a price p_j by the other seller:

$$\max_{p_i} \{(p_i - w_i)\, Q_i(p_i, p_j) - f_i\},$$

where f_i is a cost already sunk and the total number of consumers, x_0, is given from the first period. Seller i's first-order condition is

$$x_0(1 - \theta - 2\, p_i + w_i + \theta\, p_j) = 0,$$

so its equilibrium price can be easily shown to be

$$p_i(w_i, w_j) = \frac{(2 + \theta)(1 - \theta) + 2\, w_i + \theta\, w_j}{(2 + \theta)(2 - \theta)}. \tag{2}$$

It readily follows from (1) that each consumer buys

$$q_i(w_i, w_j) = \frac{(1 - \theta)(2 + \theta) - w_i \left(2 - \theta^2\right) + \theta w_j}{(1 - \theta^2)\,(4 - \theta^2)} \tag{3}$$

units of product i.

they have to decide whether to accept contracts is standard in some industries such as video games (see Hagiu, 2006, for example). If p_0 was chosen after sellers have decided to accept the platform's offers, sellers would anticipate a hold-up problem that would harm the platform. Note also that it is in principle easier to contract upon p_0 than upon other seller's fees because sellers will eventually observe p_0, but they may never be able to observe the royalty fees paid by other sellers.

We now turn to the analysis of the first period, taking into account that the utility of consumer x given p_0, p_1 and p_2 is

$$U_x(p_1, p_2, p_0) = u(p_1, p_2) - x - p_0,$$

where

$$u(p_1, p_2) = \frac{2(1-\theta)(1-p_1)(1-p_2) + (p_1 - p_2)^2}{2(1+\theta)(1-\theta)}.$$

By symmetry, optimal royalties are such that $w_1 = w_2 = w$, so

$$u^t(w) \equiv u(p_1(w,w), p_2(w,w)) = \frac{(1-w)^2}{(1+\theta)(2-\theta)^2},$$

where the superscript t refers to the two-sided, public-contract case. This results in a demand for the platform good equal to

$$x_0^t(w, p_0) = u^t(w) - p_0.$$

Anticipating how play will evolve in the second period, seller i will accept the contract offered by the platform if and only if $f_i \le x_0^t(p_i - w_i)\, q_i$. The platform provider clearly sets $f_i = x_0^t(p_i - w_i)\, q_i$, and therefore solves

$$\max_{w, p_0} \{\, x_0^t(w, p_0)\, [p_0 + p_1(w,w)\, q_1(w,w) + p_2(w,w)\, q_2(w,w)]\}. \tag{4}$$

It is then straightforward to prove the following result.

Proposition 1 (Public contracts in a two-sided market)**.** *If contracts are publicly observed by all parties, equilibrium royalties are* $w^t = -(1-\theta) < 0$*, the equilibrium price charged by seller* $i \in \{1, 2\}$ *is* $p_i^t = 0$*, the equilibrium price for the platform good is* $p_0^t = \frac{1}{2(1+\theta)} > 0$*, per-capita consumption of each product is* $q_i^t = \frac{1}{1+\theta}$*, the number of consumers who join the platform is* $x_0^t = \frac{1}{2(1+\theta)}$*, platform profits are* $\pi_0^t = \frac{1}{4(1+\theta)^2}$*, and consumer surplus is* $cs^t = \frac{1}{8(1+\theta)^2}$*.*

Note that the optimal royalty fee is always negative, and goes to zero as θ goes to one. To understand this result, we start by noting that the first-order condition of the platform provider with respect to price p_0 is

$$x_0 + \frac{\partial x_0}{\partial p_0}(p_0 + p_1 q_1 + p_2 q_2) = 0.$$

Given that the platform provider perfectly anticipates the second-period prices as a function of royalties, it can solve the problem in expression (4) as if it was choosing prices p_i instead of royalties w_i. Then it would choose price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) + \frac{\partial x_0}{\partial p_1}(p_0 + p_1 q_1 + p_2 q_2) = 0. \tag{5}$$

The first-order condition with respect to p_0 implies that

$$p_0 + p_1 q_1 + p_2 q_2 = x_0,$$

given that $\partial x_0/\partial p_0 = -1$. Since Roy's identity implies that $\partial x_0/\partial p_1 = -q_1$, the first-order condition in expression (5) becomes:

$$x_0 \left(p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

In a symmetric equilibrium,

$$x_0 \, p_1 \left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) = 0,$$

so it is optimal to induce sellers to sell their products at their marginal cost of zero. Given that seller 1 chooses price p_1 so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0,$$

the royalty fee must be negative so that sellers choose prices equal to their marginal costs. Note that prices go to marginal cost as $\theta \to 1$ due to pure Bertrand competition, so the royalty converges to zero when products become perfect substitutes. Hence the results in Proposition 1.

Finally, note that the results in this section stand in stark contrast with those of the vertical relations literature. For instance, it is well known when $\theta = 0$ that an upstream supplier would avoid double marginalization and hence would not distort pricing by a downstream retailer, which is not the case in our setting because of the presence of indirect network effects.

4. Private contracts

In this section, we assume that the contracts between the platform provider and sellers are private. Thus, consumers cannot observe any of the contracts offered to sellers, and a seller can only observe the contract it is offered. We will seek for symmetric Perfect Bayesian Equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.[8] In what follows, let p_0^* denote the price charged to consumers by the platform provider in a symmetric PBE. Also, let w^* denote the royalty fee that is offered to seller $i \in \{1, 2\}$ in a symmetric PBE, and f^* the associated fixed fee.

Regarding the formation of out-of-equilibrium beliefs, note that, upon observing any $p_0 \neq p_0^*$, rational consumers would realize that such a deviation affects sellers' profits and potentially their incentives to enter the market (this happens when $p_0 > p_0^*$). They should therefore conclude that a price deviation must be accompanied by a change in the fixed fee and/or a change in the royalty fee offered to each seller. We will look at equilibria in which consumers rationalize any price deviation by conjecturing that there was no deviation in

[8] No asymmetric equilibrium exists, so the symmetry requirement is without loss of generality, at least if one restricts attention to equilibria in which the pricing strategy and beliefs held by a seller are polynomial functions of the royalties it observes.

the royalty fee offered to each seller; hence, consumers believe upon observing $p_0 \neq p_0^*$ that the platform is simply adjusting the fixed fee offered to each seller just to make it break-even given w^*. These beliefs are in the spirit of "passive beliefs" (Hart and Tirole, 1990), but they require some rationability by consumers. In particular, when consumers observe a price deviation, they acknowledge that this should have had an impact on the sellers' willingness to accept the contract, and they reason that the absence of such an impact must be due to a change in the fixed fee offered to each seller. We refer to this weak form of passive beliefs held by consumers as "weakly passive beliefs," and note that the main implication of such belief formation is that consumers always expect the interaction of sellers in the product market to be unaffected by the choice of p_0.[9]

Because a seller anticipates such unsophisticated behavior by consumers when $p_0 \neq p_0^*$, it believes that $p_0 \neq p_0^*$ conveys no information about contract offers. Thus, sellers therefore form passive beliefs with respect to deviations in p_0. However, seller $i \in \{1, 2\}$ is assumed to form "wary beliefs" (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when it observes an unexpected contract offer $(w_i, f_i) \neq (w^*, f^*)$. In such case, it believes that the platform provider must have made an offer to j that maximizes the platform provider's total profit given the price that it charges to consumers and the contract offered to seller i. We assume as well that seller i conjectures that the other seller also forms wary beliefs, and that the platform provider does not want to drive the other seller out of the market. We emphasize that, in equilibrium, a seller perfectly anticipates the offer made by the platform to the other seller, as is usual. In turn, consumers also anticipate perfectly the contract offers made to sellers in equilibrium.

4.1. **Contracts observable to sellers, but unobservable to consumers.** Before examining equilibrium play when the contract offer received by a seller is solely observed by such a seller, it is useful to examine an intermediate case in which sellers observe each other's contract, but consumers do not. As we show next, such unobservability gives rise to a hold-up problem that consumers anticipate: consumers will (correctly) believe that the platform provider will induce sellers to charge high (collusive) prices. Sellers will earn more for each consumer who joins the platform, but the platform's value to consumers will be harmed by such beliefs. Both these forces induce the platform provider to lower access prices for buyers, thereby setting a negative markup on them.

[9] The outcome would be the same under the standard strong form of passive beliefs (corresponding to situations in which consumers do not change their equilibrium beliefs when observing out-of-equilibrium behavior). However, it would be harder to interpret some situations. For example, upon observing $p_0 > p_0^*$, a consumer who kept her beliefs about f^* and w^* should conclude that the sellers are accepting a contract that gives them negative profits, for consumer demand is smaller than it should be in equilibrium (since we shall show later on that consumer demand for the platform does not affect competition between sellers, which solely depends on royalty fees).

To see these issues formally, let us denote the contract offered to each seller in equilibrium by $(\widehat{f}, \widehat{w})$. Because consumers cannot observe deviations from this contract and form weakly passive beliefs when observing any p_0, their demand for the platform when observing price p_0 equals

$$x_0(\widehat{w}, p_0) = \frac{(1-\widehat{w})^2}{(1+\theta)(2-\theta)^2} - p_0.$$

The platform provider extracts all the surplus from sellers, and chooses p_0, w_1 and w_2 to maximize

$$x_0(\widehat{w}, p_0)\,[p_0 + p_1(w_1, w_2)q_1(w_1, w_2) + p_2(w_2, w_1)q_2(w_2, w_1)].$$

The first-order condition corresponding to w_i is

$$\frac{\theta(1-\theta)(2+\theta)^2 - (8-6\theta^2)w_i + 2\theta^3 w_j}{(1-\theta^2)(4-\theta^2)^2} = 0.$$

Rearranging this equation allows us to give it an interpretation that will be useful later on. When seller j receives an offer involving royalty fee w_j, it infers that the platform provider finds it optimal to charge seller i with a royalty fee w_i that solves the above first-order condition. Thus, given a royalty offer of w, a seller believes that the other seller is being offered a royalty equal to

$$\widehat{w}^*(w) = \frac{\theta\,(1-\theta)\,(2+\theta)^2 + 2\,\theta^3\,w}{2\,(4-3\,\theta^2)}, \tag{6}$$

Therefore, $\widehat{w}^*(\cdot)$ can be interpreted as a seller's belief about the royalty fee offered to the other seller. Such a belief is correct both on and off the equilibrium path because the platform anticipates that sellers will have complete information when pricing, so there is no way to fool them. The function $\widehat{w}^*(\cdot)$ will serve as a useful benchmark when we further assume in the next subsection that sellers cannot observe each other's contract offers.

We now complete our solution of the model. It is straightforward to show that the equilibrium royalty fee is

$$\widehat{w} = \frac{\theta}{2}.$$

Thus, the royalty fee is positive if $\theta > 0$ and negative if $\theta < 0$. The first-order condition corresponding to p_0 can be written as

$$\frac{(2-\theta)^2}{4\,(1+\theta)\,(2-\theta)^2} - \frac{1}{2\,(1+\theta)} - 2p_0 = 0,$$

so

$$\widehat{p_0} = -\frac{1}{8\,(1+\theta)} < 0.$$

In equilibrium, the platform induces seller i to charge the collusive price,

$$\widehat{p_i} = \frac{1}{2} > 0,$$

and gains

$$\widehat{\pi}_0 = \frac{9}{64\,(1+\theta)^2}.$$

To understand these results, we can proceed as in the previous section. If the platform acts as if it was choosing price p_1 instead of royalty fee w_1, it would choose price p_1 according to the following first-order condition:

$$x_0\left(q_1 + p_1\frac{\partial q_1}{\partial p_1} + p_2\frac{\partial q_2}{\partial p_1}\right) = 0.$$

Note that this first-order condition differs from the one in Section 3 because consumers do not observe changes in royalty fees, so their decision to buy the platform good depends only on their *beliefs* about the equilibrium royalty. In a symmetric equilibrium, it holds that

$$-\left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1}\right)p_i = q_i.$$

Thus, the optimal implied price for sellers is positive. This contrasts with the result in the public-contract case, in which the optimal price was zero.

It is easy to see that the optimal price p_0 solves

$$p_0 = \frac{u(p_1,p_2) - (p_1q_1 + p_2q_2)}{2}.$$

This equation shows that the platform has incentives to lower p_0, in comparison with the public-contract case, for two reasons: to compensate the decrease in consumer surplus from consumption of seller goods ($u(p_1,p_2) < u(0,0)$), and because seller surplus per consumer increases ($p_1q_1 + p_2q_2 > 0$). In the case at hand, it turns out that the platform lowers p_0 so much that it ends up setting a negative access fee for consumers.

Finally, note that seller i chooses price p_i so that

$$x_0\left(q_1 + p_1\frac{\partial q_1}{\partial p_1} - w_1\frac{\partial q_1}{\partial p_1}\right) = 0.$$

Thus, the royalty needs to be positive if the cross-price effect $\partial q_2/\partial p_1$ is positive (substitutes), and negative if the cross-price effect is negative (complements).

Summarizing, we find that when consumers do not observe royalty fees, they are less reactive to changes in the intensity of competition between sellers, since they cannot observe deviations from the royalty fees they expect in equilibrium. As a consequence, the platform has incentives to behave opportunistically, and choose royalties to induce collusive pricing by sellers. Consumers correctly foresee the hold-up problem when they decide whether to join the platform, so their utility from joining the platform decreases. The platform has incentives to lower access prices for consumers for two reasons: to compensate the lower demand for platform access, and because seller revenue per consumer increases.

4.2. **Contracts unobservable to sellers and consumers.** We now turn to the analysis of the private contracts case, in which the contract offer received by a seller is solely observed by this seller.

At the beginning of the second period, seller $i \in \{1,2\}$ observes p_0, x_0, f_i and w_i, and chooses a price for its product based on this information. Taking into account that seller i's overall demand product equals $Q_i(p_i, p_j) = x_0 q_i(p_i, p_j)$, we can solve for the second-period subgames.

Let $B(\widehat{w})$ denote the belief formed by seller i about the royalty fee paid by seller j to the platform provider.[10] We follow Rey and Vergé (2004), and restrict attention to equilibria in which seller i's belief about the royalty fee paid by the other seller does not depend on the fixed fee it observes. Not only is the pricing strategy of seller i independent from the fixed fee it already paid, but it is also independent from p_0 (and hence from x_0). Such a price has no signaling role and it does not affect belief formation, which seems a reasonable assumption given that x_0 is simply a scaling factor in seller i's second-period profit.[11]

In what follows, let $p_i(w_i)$ denote the strategy of seller $i \in \{1,2\}$ in the second-period subgame if it has observed an offer of (w_i, f_i) and price p_0. Seller $i \in \{1,2\}$ chooses p_i to maximize

$$(p_i - w_i)Q_i(p_i, p_j(B(w_i))) - f_i,$$

with f_i already sunk. The first-order condition is

$$1 - \theta + w_i - 2p_i(w_i) + \theta p_j(B(w_i)) = 0. \tag{7}$$

We now turn to analyzing the first period of play. Regardless of the price p_0 that consumers observe, they believe that both sellers face a royalty fee w^*, so they expect a price

$$p_i^* = \frac{1 - \theta + w^*}{2 - \theta}$$

for each unit they purchase from seller $i \in \{1,2\}$ in the second period. Given price p_0, the overall utility expected by consumer x is

$$U_x(w^*, p_0) = \frac{(1 - w^*)^2}{(1+\theta)(2-\theta)^2} - x - p_0,$$

[10]Because we are looking at symmetric equilibria, the belief function $B(\cdot)$ does not depend on the label of the seller receiving the possibly unexpected offer. Note that, in general, $B(\cdot)$ is an unrestricted function except for the constraint that $B(w^*) = w^*$ (i.e., conjectured beliefs are fulfilled along the equilibrium path). In our case, we restrict the function so that beliefs be wary, but we also consider later on what happens if beliefs are passive (i.e., $B(w) = w^*$ for all w).

[11]Therefore, it does not affect equilibrium pricing in the second-period if sellers believe that it does not convey some information, making it self-fulfilling that it is pointless for the platform provider to use it for signaling purposes.

so the demand for the platform good is

$$x_0(w^*, p_0) = \frac{(1-w^*)^2}{(1+\theta)\,(2-\theta)^2} - p_0.$$

The platform provider's total profit if it charges p_0 and makes a private offer of (w_1, f_1) and (w_2, f_2) to sellers 1 and 2 is

$$\begin{aligned}\pi_0(w_1, f_1, w_2, f_2, p_0) \quad &= \quad x_0(w^*, p_0)\,[p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) \\ &\quad + w_2\, q_2(p_2(w_2), p_1(w_1))] + f_1 + f_2,\end{aligned}$$

since the platform provider can perfectly forecast actual sales made by sellers 1 and 2. In order for seller 2 (say) to form wary beliefs, her inference about seller 1's contract upon observing a price of p_0 and an offer of (w_2, f_2) must be such that $B(w_2)$ maximizes $\pi_0(w, f, w_2, f_2, p_0)$ with respect to w and f subject to the constraint that

$$f \leq (p_1(w) - w)x_0(w^*, p_0)q_1(p_1(w), p_2(B(w))).$$

Taking into account that the constraint must bind at the optimum, and that (7) implies

$$q_1(p_1(w), p_2(B(w))) = \frac{p_1(w) - w}{1-\theta^2},$$

we have

$$B(w_2) \in \operatorname*{argmax}_w \pi_0'(w, w_2, f_2, p_0), \tag{8}$$

where

$$\begin{aligned}\pi_0'(w, w_2, f_2, p_0) \quad &= \quad x_0(w^*, p_0)\Bigg\{p_0 + w\, q_1(p_1(w), p_2(w_2)) \\ &\quad + w_2\, q_2(p_2(w_2), p_1(w)) + \frac{[p_1(w) - w]^2}{1-\theta^2}\Bigg\} + f_2.\end{aligned}$$

The first-order condition with respect to w is

$$\begin{aligned} q_1(p_1(w), p_2(w_2)) + \frac{2[p_1(w) - w]}{1-\theta^2}\left(\frac{dp_1(w)}{dw} - 1\right) \\ + \left[w\frac{\partial q_1(p_1(w), p_2(w_2))}{\partial p_1} + w_2\frac{\partial q_2(p_2(w_2), p_1(w))}{\partial p_1}\right]\frac{dp_1(w)}{dw} \quad = \quad 0.\end{aligned} \tag{9}$$

Since our purpose at this stage is to build some intuition, let us assume for now that a unique solution to equation (9) exists for any w_2. Let $w_1^*(w_2)$ denote that solution, and note that it must coincide with $B(w_2)$ even if $w_2 \neq w^*$ because sellers form wary beliefs when off the equilibrium path. Using the implicit function theorem, we obtain the following result:

$$\frac{dB(w_2)}{dw_2} = \frac{dw_1^*(w_2)}{dw_2} = -\frac{\dfrac{\theta}{1-\theta^2}\left(\dfrac{dp_2(w_2)}{dw_2} + \dfrac{dp_1(w)}{dw}\right)}{\partial^2\pi_0'(w, w_2, f_2, p_0)/\partial w^2}.$$

If $\pi_0'(w, w_2, f_2, p_0)$ is strictly concave with respect to w (as we shall later show), symmetry yields that

$$\text{sign}\left(\frac{dB(w)}{dw}\right) = \text{sign}\left(\theta\frac{dp(w)}{dw}\right).$$

Whenever it holds that $dp(w)/dw > 0$, which is an intuitive property that equilibrium prices should satisfy,[12] we have that $dB(w)/dw \gtreqless 0$ if and only if $\theta \gtreqless 0$, according well with what one may have expected: sellers' prices are *strategic complements* if $\theta > 0$ and *strategic substitutes* if $\theta < 0$, and the platform provider aims at softening competition between sellers under strategic complementarity and at toughening such competition under strategic substitutability.

Having shed some light on some of the properties that the equilibrium satisfies, we proceed to showing existence and characterizing it. To this end, evaluating the first-order condition in expression (9) at $w = B(w_2)$, and letting $p_i(w) = p(w)$ because of symmetry, we obtain

$$\begin{aligned} 0 \quad = \quad & 1 - \theta - p(B(w_2)) + \theta p(w_2) + (\theta w_2 - B(w_2))\frac{dp(B(w_2))}{dw} \\ & +2[p(B(w_2)) - B(w_2)][\frac{dp(B(w_2))}{dw} - 1]. \end{aligned} \tag{10}$$

If one focuses on PBE such that $p(\cdot)$ and $B(\cdot)$ are polynomial functions, then Rey and Vergé (2004) show that there is no loss of generality in restricting attention to affine functions, so one can readily solve the system of differential equations given by (10) and (7) (after dropping subscripts). In Appendix A, we prove the following result.

Proposition 2 (Private contracts in a two-sided market)**.** *The unique symmetric PBE in which $p(w)$ and $B(w)$ are polynomial functions is such that $p(w) = \Theta_\theta + \Sigma_\theta w$ and $B(w) = \Gamma_\theta + \Phi_\theta w$ for some constants $\Theta_\theta \in [0, 1]$, $\Sigma_\theta \in [\frac{1}{2}, 1] > 0$, $\Gamma_\theta \in [0, 1]$, and $\Phi_\theta \in [-1, 1]$. In such an equilibrium, it always holds that*

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \geq 0$$

for $i \in \{1, 2\}$ as well as that $p_0^* < 0$ and $w^* \geq 0$ for any $\theta \in (-1, 1)$, with $w^* = 0$ if and only if $\theta = 0$. Platform profits are

$$\pi_0^* = \left(\frac{1 - p_i^{*2}}{2\,(1 + \theta)}\right)^2,$$

and consumer surplus is

$$cs^* = \frac{1}{2}\left(\frac{1 - p_i^{*2}}{2\,(1 + \theta)}\right)^2.$$

[12] Note that we shall restrict attention to polynomial pricing strategies, and that in such cases there is no loss in further restricting them to be affine.

Contrary to the case in which sellers can observe each other's royalties (subsection 4.1), royalty fees are never negative under private contracting, regardless of whether price competition between sellers displays strategic complementarity ($\theta > 0$) or strategic substitutability ($\theta < 0$).

When sellers can observe each other's royalties, $\theta > 0$ implies that $d\widehat{w}^*(w)/dw > 0$ (see expression (6)), so an increase in the royalty fee a seller observed would (correctly) make it believe that the other seller's royalty offer must have increased, since the platform aims at softening competition, and hence in equilibrium $\widehat{w} = \theta/2 > 0$. The converse happens if $\theta < 0$ (so that $d\widehat{w}^*(w)/dw < 0$), with $\widehat{w} = \theta/2 < 0$ in these cases because the platform wishes to toughen competition. When sellers *cannot* observe each other's offers, their beliefs become more sensitive to observed royalties. This overreaction to changes in the royalty fee observed is a straightforward effect of wary beliefs.

Figure 1 plots $d\widehat{w}^*(w)/dw$ (see solid curve) relative to $dB(w)/dw$ (see dashed curve) as parameter θ varies, and shows that beliefs become more sensitive to changes in the offer received from the platform when sellers cannot observe each other's contracts.

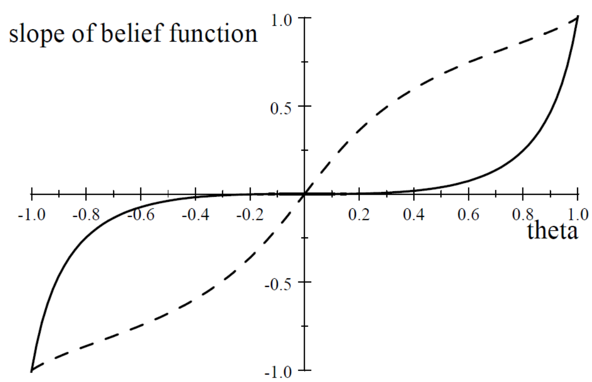

FIGURE 1. Comparison of beliefs

The determinants of how the equilibrium royalty fee relates to θ are different when sellers can observe each other's royalty offers and when they cannot. When they can observe them as in Section 4.1, the platform's incentives to deviate have to do with making competition between sellers softer (if $\theta > 0$) or tougher (if $\theta < 0$), as we just mentioned. When sellers cannot observe each other's royalty offers, the platform's incentives to deviate greatly depend on how a seller that receives an unexpected offer updates her beliefs about the royalties of the other seller. In particular, such a seller (correctly) infers that the platform provider must be simultaneously deviating with the other seller in a way that the opportunistic platform does not care about seller 2's profitability. Indeed, taking into account that the platform extracts all the surplus that seller i expects to make when observing royalty fee w_i, it holds that the payoff to the platform if it chooses w_1, w_2 and p_0 equals

$$\begin{aligned}\widehat{\pi}_0(w_1, w_2, p_0) &= x_0(w^*, p_0)\Big\{p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1))\\ &\quad + [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(B(w_1)))\\ &\quad + [p_2(w_2) - w_2]\, q_2(p_2(w_2), p_1(B(w_2)))\Big\}\\ &= x_0(w^*, p_0)\Big\{p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1))\\ &\quad + [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(w_2))\\ &\quad + [p_1(w_1) - w_1]\, \big[q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))\big]\\ &\quad + [p_2(w_2) - w_2]\, q_2(p_2(w_2), p_1(B(w_2)))\Big\}.\end{aligned}$$

Clearly, maximizing this payoff with respect to w_1 is equivalent to maximizing

$$\begin{aligned}\widehat{\pi}_0'(w_1, w_2) &= [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(w_2))\\ &\quad + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1))\\ &\quad + [p_1(w_1) - w_1]\, \big[q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))\big],\end{aligned}$$

so the platform cares about seller 1's actual profit, the actual royalty revenue generated by each seller and the change in seller 1's profit because of the formation of wary beliefs. By the envelope theorem, seller 1's actual profit when w_1 changes by a small amount is $-q_1(p_1(w_1), p_2(w_2))$, so $w^* = B(w^*)$ implies that

$$\left.\frac{\partial \widehat{\pi}_0'(w_1, w_2)}{\partial w_1}\right|_{w_1 = w_2 = w^*} = 0$$

is equivalent to

$$\left\{[p(w^*) - w^*]\, \theta\, \frac{dB(w^*)}{dw} - (1 - \theta)\, w^*\right\} \frac{dp(w^*)}{dw} = 0.$$

The fact that $p(w^*) > w^*$ then implies that

$$w^* = \frac{\theta}{1 - \theta}\, \frac{dB(w^*)}{dw}\, [p(w^*) - w^*]$$

must be nonnegative because we showed earlier that $\theta(dB(w^*)/dw) \geq 0$.

As we have shown, the sign of w^* depends on how the second argument of $q_1(p_1(w_1), p_2(B(w_1)))$ varies with w_1, that is, on whether an increase in w_1 will stimulate seller 1's sales via the conjectured price change performed by seller 2. Because seller 1 always believes that this is indeed the case, w^* is always nonnegative.

When sellers can observe each other's offer, we showed in Section 4.1 that the equilibrium royalty fee is positive if and only if competition between sellers displays strategic complementarity. Figure 2 compares royalty fees in the two models (the dashed curve corresponds to the case of private contracting).

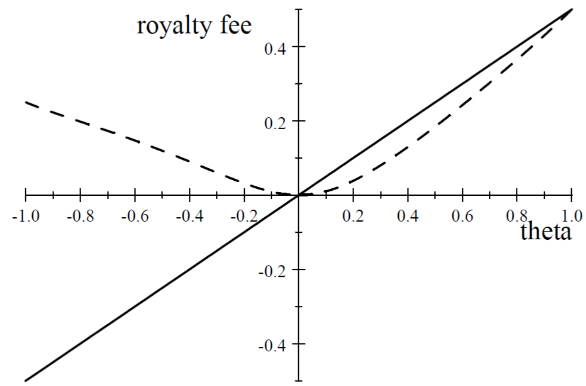

FIGURE 2. Comparison of royalty fees

Because $w^* < \widehat{w}$ if and only if $\theta > 0$, it should come as no surprise that the comparison of sellers' prices in both situations is as illustrated in Figure 3 (the dashed curve corresponds to the case of private contracting).

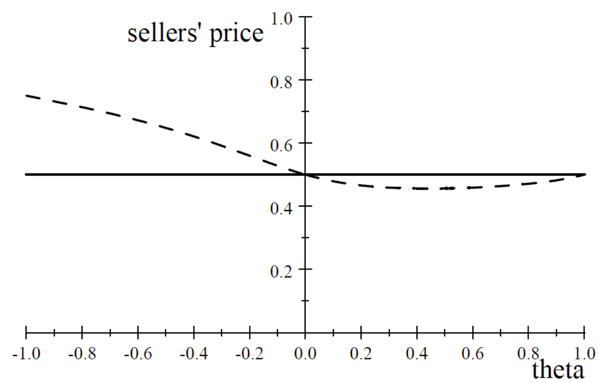

FIGURE 3. Comparison of seller prices

Relative to when sellers can observe each other's contract, it holds when they cannot that the platform provider loses part of its market power vis-à-vis sellers because of its opportunistic behavior when dealing with each on a one-on-one basis (as in Rey and Vergé, 2004). This smaller market power implies that the platform provider cannot sufficiently raise sellers' prices through the royalty fees when goods are substitutes; when goods are complements, the smaller market power of the platform provider implies that it cannot sufficiently lower prices charged by sellers so as to mitigate the double marginalization problem first pointed out by Cournot (1838) for the case of perfect complements.

The difference in pricing by sellers illustrated by the previous figure has key implications for platform pricing, since one of the two determinants of platform demand is how much utility consumers expect to attain given the anticipated pricing by sellers. When $\theta < 0$, consumers correctly anticipate that sellers will charge higher prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to lower the platform's price relative to when sellers can observe each other's offer. When $\theta > 0$, the sellers charge lower prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to raise the platform's price relative to when sellers can observe each other's offer.

The other determinant of platform pricing is how much overall profit is generated per consumer through the two sellers. Figure 4 shows how total profit generated by sellers per customer varies with θ (the dashed curve represents the situation when seller cannot observe each other's offer).

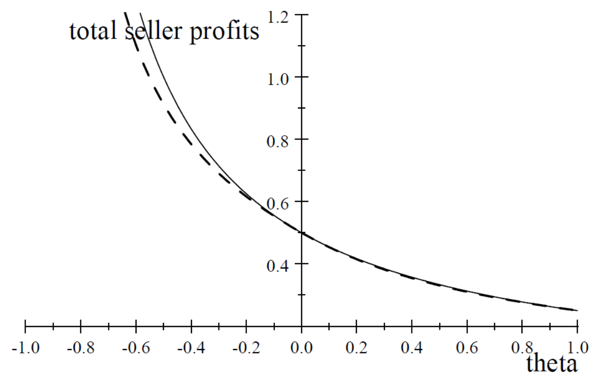

FIGURE 4. Comparison of seller profits per customer

Because sellers are induced to price collusively when they can observe each other's offer, it holds that per-consumer profitability is at least as large as when they cannot observe each other's offer. This implies that, regardless of the value of θ, the platform provider has an incentive to set a higher price for the platform when sellers cannot observe each other's offer than when they can. Interestingly, note that the incentive is very small when $\theta > 0$: in such cases, the platform provider's opportunistic behavior is hardly costly in terms of generating sellers' profits. The effect highlighted by Rey and Vergé (2004) is present, but it is not very strong.

Overall, we find that pricing by the platform is mostly driven by the anticipated effect of sellers' prices on consumer utility. On the one hand, when $\theta > 0$, the platform provider prices higher when sellers cannot observe each other's offer than when they can: the effect on consumer demand of having lower prices dominates the effect of appropriating less profit through sellers. On the other hand, when $\theta < 0$, the effect of having lower consumer utility when sellers cannot observe each other's offer always dominates the lower per-consumer profitability that arises when sellers cannot observe each other's offer. This is illustrated by Figure 5 (the dashed curve corresponds to the case of private contracting).

It should then not be very surprising that platform profits are greater when sellers cannot observe each other's offer than when they can if and only if $\theta > 0$, as Figure 6 shows (the dashed curve represents the situation when seller cannot observe each other's offer). A

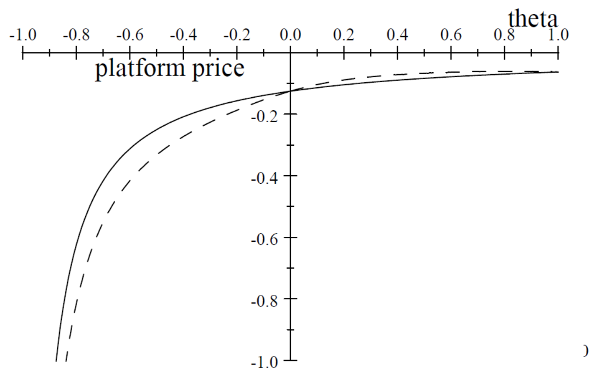

FIGURE 5. Comparison of consumer access prices

similar result holds for consumer and total welfare, since they are proportional to platform profits both if sellers can or cannot observe each other's offer.

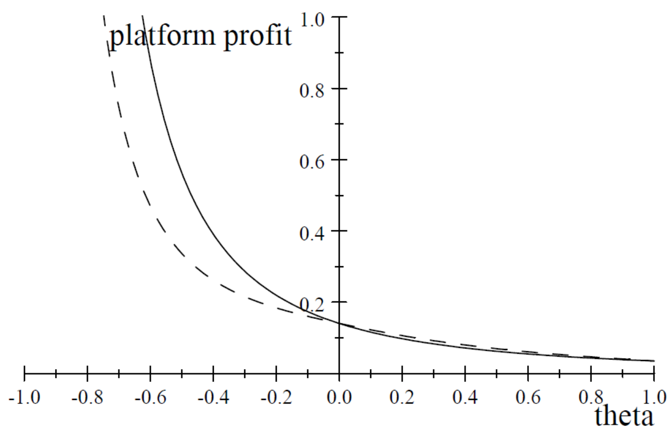

FIGURE 6. Comparison of platform profit

We now turn to our main result. The following proposition shows the effects of private contracts in a two-sided market by comparing the equilibrium of this section with that of the previous two sections.

Proposition 3 (Implications of private vs. public contracts)**.** *Equilibrium royalties are negative with public contracts, and are positive with private contracts. The price of the platform good for consumers is positive with public contracts, and is negative with private contracts. Private contracts lead to lower profit, consumer surplus, and welfare.*

The first two claims in the proposition follow from comparing the equilibria of the models in Sections 3 and 4. The proof for the last claim is included in the proof of Proposition 2.

5. Private contracts with a large number of sellers

An interesting case to examine is that in which n becomes very large and strategic interaction among sellers becomes very weak. If $\theta > 0$, sellers compete monopolistically and the platform ends up having an absolute control over sellers' prices through royalty fees. As $n \to \infty$ in these cases, both $p(w)$ and $B(w)$ converge pointwise to w, whereas w^* and p^* both converge to 1/2 for $\theta > 0$.[13] The platform provider therefore has no incentive to act opportunistically with respect to any seller and it induces collusive pricing by sellers in equilibrium, thus depressing consumers' incentives to join the platform as in Section 4.1. This gives a rationale for the platform provider to limit entry by sellers into the platform: when the platform provider can act opportunistically, it prefers that there be strategic interaction among sellers so as to commit to control less their pricing and thus alleviate the consumers' hold-up problem. Restricting massive market participation by sellers would not only benefit the platform but also consumers, so it would be welfare-enhancing if love-for-variety effects are left aside.

6. Policy implications

In this section, we study alternative policies that may be used by the platform to mitigate the adverse effects of private information. We will show that the key to understanding the effects of the different policies is to determine how they deal with the various informational problems.

6.1. **Most favored nation and price-forcing clauses.**

Contracts between platform and sellers in two-sided markets often use Most Favored Nation (MFN) and price-forcing clauses. For example, the private contract between Spotify (an online music platform) and Sony Music (a record company) included MFN clauses,[14] and seller prices are regulated by the platform in the case of iTunes. We next study the welfare implications of each instrument.

Suppose first that it is commonly known that the platform provider includes MFN clauses in their contracts with sellers. Contracts are still private for other sellers and consumers, but sellers are guaranteed not to obtain worse terms than other sellers. We model this situation

[13] All these arguments can be shown formally, and the proof is available upon request. We note that $p(w) = (1+w)/2$ and $B(w) = 0$ when $\theta = 0$, so $w^* = 0$ and $p^* = 1/2$ when sellers' products are independent. Therefore, there is a discontinuity at $\theta = 0$ in terms of pricing strategies, belief functions and royalties, but not in terms of the price that sellers are induced to charge.

[14] The contract reads: "Company [Spotify] represents, warrants and covenants that, as of the date of execution of this Agreement and throughout the remainder of the Term, it has not entered into any agreement or other understanding with any third party provider of recorded music, videos artwork or other assets used in connection with the Services (each, an "Other Party") that, includes terms and conditions which, when evaluated as a whole, are more favorable to such Other Party than the terms and conditions, when evaluated as a whole, contained in this Agreement." Contracts such as this one are generally private. However, in the case at hand, the contract was made public by hackers that entered into Sony's servers. See http://www.nytimes.com/2015/05/25/business/media/sony-terms-with-spotify-uncovered-in-contract.html, accessed October 7, 2016, for more details.

by assuming that sellers have "symmetry beliefs" (McAfee and Schwartz, 1994). That is, sellers believe that $B(w) = w$ for all w.

It is straightforward to see that with this type of beliefs, results are identical to those of Section 4.1. Therefore, the introduction of MFN clauses makes the platform and consumers worse off when goods are substitutes, but it makes all of them better off when goods are complements. The reason is that MFN clauses solve sellers' information problems, but not consumers', which intensifies the hold-up problem when sellers' products are substitutes, and alleviates it when they are complements.[15]

Proposition 4 (Most favored nation clauses)**.** *Relative to when MFN clauses are not used, the introduction of such clauses increases total welfare if sellers' products are complements and decreases total welfare if they are substitutes.*

Suppose now that the platform includes price-forcing clauses in its contract with sellers, so that sellers commit to sell to consumers at a contract-specified price. In contrast with MFN, price-forcing clauses can solve the informational problems of consumers if the platform includes a commitment to maintain sellers' prices at a certain level in its contracts with consumers.

The best the platform can do is to replicate the results in Section 3 in order to make the consumers' hold-up problem disappears, thereby alleviating consumers' fears when adopting the platform, so it is dominant for the platform to induce marginal cost pricing by sellers. Doing so makes not only the platform better off, but also consumers, which leads to our next result.

Proposition 5 (Price-forcing contracts)**.** *Relative to when price-forcing clauses are not used, the introduction of such clauses always increases total welfare.*

The reason for this result is that, in contrast with MFN clauses, this type of Resale Price Maintenance (RPM) clause solves not only sellers' information problems, but also consumers', and can thus be used to replicate the equilibrium of the public-contract case.

Our results suggest that price-forcing contracts are advantageous to the platform and to consumers as well because they allow the platform to commit to low seller prices and stimulate platform adoption. This result may explain why price-forcing contracts are common in two-sided markets, as in the cases of iTunes and Uber.

In the case of Sony's PlayStation Plus platform, for example, gamers pay a yearly fee for access to free or discounted games. Developers negotiate PS Plus contracts with Sony on

[15] Sony's contract with Spotify included a clause by which Sony had the right to "request that an independent third party auditor" determine whether Spotify had "entered into a potentially Generally More Favorable Agreement with any Other Party." Therefore, even though Sony did not have access to Spotify's contracts with other sellers, it could be sure that those contracts did not include more favorable terms than its own contract. Nevertheless, contracts remained secret for consumers, and thus did not solve their informational problems.

a case-by-case basis, and agree on the price at which they will sell games on the platform. Sony, in turn, commits to having a large number of games at zero or discounted price available for PS Plus' subscribers, which provides insurance to consumers about future game prices.[16]

To the best of our knowledge, ours is the first paper providing a rationale for the use of price-forcing clauses in a two-sided market setting. Use of such clauses can be welfare-enhancing in cases of dominant platforms, as in the cases of iTunes, Uber and PS Plus, regardless of whether there are many or few sellers. We also note that this result stands in stark contrast with that found in vertical relations settings. In such cases, price-forcing contracts are well-known to damage welfare even if they are profitable for an upstream monopolistic supplier (see e.g. the excellent survey by Rey and Tirole, 2007).

6.2. **Agency vs. wholesale models of sales.** Our model can also shed some light on the private and social desirability of what is known as the "agency model of sales" (Johnson, 2014), which is used, for example, by Apple for selling e-books on its platform. In the agency model of sales, the platform acts as a sales agent of sellers, charging a fee for selling the sellers' goods through its platform, and sellers still set the price at which they want to sell their goods.[17] In terms of our model, the agent (platform) privately offers a two-part tariff (f_i, w_i) to seller i, and this seller chooses price p_i for its goods. Therefore, the agency model of sales is equivalent to the two-sided platform studied in Section 4 and leads to similar informational problems.

The alternative to the agency model of sales is the wholesale, merchant, or retail model, used for example by Amazon for selling e-books before 2010. In the wholesale or merchant mode, the platform buys the goods from sellers, and sells them directly to consumers. In our setting, this corresponds to sellers becoming upstream suppliers of the platform, and to the platform becoming a one-sided downstream retailer, selling the platform and sellers' goods to consumers. Because the platform can include prices for sellers' goods in its contract with consumers, and given that the platform has all the bargaining power when it negotiates with sellers, this model is equivalent to the one in which the platform uses price-forcing contracts studied in the previous section. We have the following result.

Proposition 6 (Agency vs. wholesale models of sales)**.** *Platform profits and consumer welfare are both higher when the platform controls sellers' prices directly than when it simply controls their royalty fees, so the wholesale model is privately and socially preferred over the agency model of sales.*

[16]See http://www.gamasutra.com/view/feature/191966/, accessed October 7, 2016, for more details.

[17]The agency model of sales is a business model that should not be confused with the principal-agent model studied in information economics.

It is important to note that in the setting studied in this paper, the only difference between the two models is how they deal with informational problems. If all contracts are observed, then both models yield equivalent results. The only reason why the wholesale model outperforms the agency model in our paper is because it allows the platform provider to become more transparent in its dealings with consumers, making informational frictions vanish.

The contrast between the agency and wholesale models has recently been made relevant by Amazon's conflicts with book publishers, in particular with French publisher Hachette. Book publishers argued that Amazon was setting too low downstream prices for books, which forced them to accept low upstream prices from Amazon. Apple, in contrast, allowed publishers to set their own prices, and therefore, publishers pushed for Amazon to embrace the agency model, which it did between January and March of 2010.[18]

Our model gives support to Amazon's claims in its fight with publishers, because it shows that the wholesale model is better apt for solving consumers' informational problems.[19] However, it is worth remarking that in our setting the platform provider makes take-it-or-leave-it offers to sellers when bargaining over fees or prices, so sellers' profits are zero in either model (agency or wholesale). To properly understand publishers' incentives, we would need to account for sellers' bargaining power, which we believe is an interesting direction for further research.

Our model also contributes to the recent discussion over a platform provider's choice between the merchant one-sided and two-sided platform business models (Hagiu, 2007; Hagiu and Wright, 2014, 2015, 2016). We show that informational problems can help explain why many platforms operate as one-sided platforms, when they could instead be run as two-sided platforms, as in the cases of Amazon and Zappos.[20] Similarly, our paper helps explaining why firms such as Netflix, Pandora, and Hulu choose to use subscription models with positive membership fees and zero transaction fees, which exactly replicates the price structure of Section 3.

6.3. **Integration between platform and sellers.** A natural alternative to contracting with sellers is to acquire them. When the platform provider is integrated with both sellers, we compare two situations, depending on whether or not the platform provider is able (or willing) to commit to the prices charged for sellers' goods when consumers get access to the platform.

[18]For these actions, the US Department of Justice sued Apple and publishers Hachette, Harper Collins, Macmillan, Penguin, and Simon & Schuster, for conspiring to raise the price of e-books in violation of the Sherman Act. See United States of America v. Apple Inc., et al., 12 Civ. 2862 (DLC) for details.

[19]In its suit against Apple and book publishers, the DOJ presented evidence that book sales fell significantly after Amazon changed its business model, which is consistent with our results.

[20]Interestingly, Amazon operates several business models. For example, Amazon works under the merchant mode for selling products directly to consumers, operates as a two-sided platform when sellers sell goods through Amazon Marketplace, and uses a combination of the merchant and two-sided platform modes for selling music and movies.

Commitment may arise when the contract signed between the platform provider and the consumers includes not only the access price, but also the prices charged by sellers 1 and 2. Clearly, under such type of commitment, all informational frictions disappear and the situation such as the one in Section 3 arises, so a move towards complete integration would be a Pareto-improvement.

When a fully integrated platform provider cannot commit to the prices charged for sellers' goods, on the other hand, the consumers and the platform provider face a situation identical to that studied in Section 4.1. Therefore, the lack of commitment lowers platform profits and social welfare, relative to when commitment is feasible.

We can also compare integration in the absence of commitment with situations in which the platform provider is not integrated and privately contracts with each seller. In such case, it holds that integration is (privately and socially) preferred over non-integration if and only if $\theta < 0$ (see Figure 6). We have thus the following result.

Proposition 7 (Platform integration with sellers)**.** *Under commitment, integration is privately and socially desirable. In the absence of commitment, integration is privately and socially desirable if and only if sellers' goods are complements.*

Proposition 7 contrasts with the findings of previous papers studying vertical relations in one-sided markets, which show that secret contracting is socially preferred over integration. In the case of two-sided markets, in contrast, vertical integration can improve welfare because it helps firms overcome informational problems that cause inefficient hold-up problems.

7. Formation of (weakly) passive beliefs by sellers

In this section, we examine the role of belief formation on our conclusions on social welfare and price structures. We assume that sellers form (weakly) passive beliefs (Hart and Tirole, 1990; O'Brien and Shaffer, 1992) rather than wary beliefs when observing an unexpected contract offer.

Relative to the cases in which sellers observe contract offers, sellers anticipate that their interpretation of out-of-equilibrium play may allow the platform provider to take advantage of them more easily. Indeed, an offer involving a high royalty fee when $\theta > 0$ must be associated to a substantially lower fixed fee, reason why the platform provider is led to lower the royalty fee relative to when seller can observe contract offers. When $\theta < 0$, the situation is the converse, so the platform provider is led to increase the royalty fee relative to when seller can observe contract offers.

To formalize these insights, suppose that, in a symmetric PBE sustained by passive beliefs, the platform must charge $\widetilde{p}_0$ and must offer contract $(\widetilde{f}, \widetilde{w})$ to each seller. Consider, however, what happens if one of them denoted $i \in \{1, 2\}$ observes p_0 and (f_i, w_i), where p_0, f_i and w_i

need not coincide with $\widetilde{p}_0$, $\widetilde{f}$ or $\widetilde{w}$, and let us examine what happens if such a seller forms passive beliefs. The seller anticipates that the other seller must charge

$$\widetilde{p} = \frac{1-\theta+\widetilde{w}}{2-\theta},$$

so seller i anticipates charging

$$\widetilde{p}_i(w_i) = \frac{1-\theta+w_i+\theta\widetilde{p}}{2}.$$

Seller i also anticipates that the consumers' demand for the platform must equal

$$x_0(p_0,\widetilde{w}) = \frac{(1-\widetilde{w})^2}{(1+\theta)(2-\theta)^2} - p_0.$$

Regardless of what it observes, seller i believes that the other seller is offered $\widetilde{w}$ and hence expects seller i to observe equilibrium play and thus charge $\widetilde{p}$. As a result, the profit of the platform provider if it charges p_0 to consumers and it offers (f_i, w_i) to seller $i \in \{1,2\}$ is

$$\widetilde{\Pi}(w_1,w_2,p_0) = x_0(p_0,\widetilde{w})\left[\begin{array}{c} p_0 + w_1\left(\dfrac{1-\theta-\widetilde{p}_1(w_1)+\theta\widetilde{p}_2(w_2)}{1-\theta^2}\right) \\ +w_2\left(\dfrac{1-\theta-\widetilde{p}_2(w_2)+\theta\widetilde{p}_1(w_1)}{1-\theta^2}\right)\end{array}\right] + f_1 + f_2,$$

where

$$f_i = x_0(p_0,\widetilde{w})(\widetilde{p}_i(w_i)-c-w_i)\left(\frac{1-\theta-\widetilde{p}_i(w_i)+\theta\widetilde{p}}{1-\theta^2}\right),\ i=1,2.$$

Equivalently,

$$\widetilde{\Pi}(w_1,w_2,p_0) = x_0(p_0,\widetilde{w})\left[\begin{array}{c} p_0 + \sum_{i=1}^2\sum_{j=1;j\neq i}^2 w_i\left(\dfrac{1-\theta-\widetilde{p}_i(w_i)+\theta\widetilde{p}_j(w_j)}{1-\theta^2}\right) \\ +\sum_{i=1}^2(\widetilde{p}_i(w_i)-c-w_i)\left(\dfrac{1-\theta-\widetilde{p}_i(w_i)+\theta\widetilde{p}}{1-\theta^2}\right)\end{array}\right].$$

It is clearly optimal for the platform provider to deviate in the same way with respect to the royalty fees, so letting $w_i = w$ implies that its payoff function can be written as follows:

$$\widetilde{\Pi}(w,p_0) = \left(\frac{(1-c-\widetilde{w})^2}{(1+\theta)(2-\theta)^2}-p_0\right)\left[p_0 + w\left(\frac{1-c+\theta-w-\theta\widetilde{p}}{1+\theta}\right) + \frac{(1-c-\theta-w+\theta\widetilde{p})^2}{2(1-\theta^2)}\right].$$

The first-order condition with respect to w yields that

$$\left(\frac{(1-c-\widetilde{w})^2}{(1+\theta)(2-\theta)^2}-p_0\right)\left[\frac{c\theta+\theta(1-\theta)+(2\theta-1)\widetilde{w}-\theta(2-\theta)\widetilde{p}}{1-\theta^2}\right] = 0.$$

Because the solution to such first-order condition must be $w = \widetilde{w}$, one obtains that $\widetilde{w} = 0$, so the platform provider does not distort pricing by sellers (this result is in line with Rey and Vergé, 2004). Taking this result into account, the first-order condition with respect to

p_0 is

$$\frac{(1-c)^2}{(1+\theta)(2-\theta)^2} - 2p_0 - \frac{[1-c-\theta+\theta(\frac{1+c-\theta}{2-\theta})]^2}{2(1-\theta^2)} = 0.$$

Therefore,

$$\widetilde{p}_0 = -\frac{(1-2\theta)(1-c)^2}{2(1+\theta)(2-\theta)^2},$$

so the platform is sold at a loss to consumers, as when sellers form wary beliefs.

It simply remains to verify that $\widetilde{w}$ and $\widetilde{p}_0$ globally maximize $\widetilde{\Pi}(w,p_0)$. Since

$$\left.\frac{\partial\widetilde{\Pi}(w,p_0)}{\partial w}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0} = \left.\frac{\partial\widetilde{\Pi}(w,p_0)}{\partial p_0}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0} = 0,$$

we simply need to check that

$$\begin{aligned} 0 &\leq \left(\left.\frac{\partial^2\widetilde{\Pi}(w,p_0)}{\partial p_0^2}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0}\right)\left(\left.\frac{\partial^2\widetilde{\Pi}(w,p_0)}{\partial w^2}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0}\right) - \left(\left.\frac{\partial^2\widetilde{\Pi}(w,p_0)}{\partial w\partial p_0}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0}\right)^2 \\ &= (-2)\frac{(1-2\theta)[\widetilde{p}_0(1+\theta)(2-\theta)^2-(1-c)^2]}{(1-\theta)(1+\theta)^2(2-\theta)^2}, \end{aligned}$$

where the last equality makes use of the fact that

$$\left.\frac{\partial^2\Pi(w,p_0)}{\partial w\partial p_0}\right|_{w=\widetilde{w},p_0=\widetilde{p}_0} = 0.$$

As a result of straightforward algebra, $\widetilde{w}$ and $\widetilde{p}_0$ globally maximize $\widetilde{\Pi}(w,p_0)$ if and only if $\theta \leq 1/2$. This is exactly the necessary and sufficient existence condition obtained by Rey and Vergé (2004) for the case of passive beliefs in one-sided markets. In Appendix B, we show the following result.

Proposition 8 (Passive beliefs by sellers)**.** *An equilibrium in which sellers form passive beliefs (uniquely) exists if and only if $\theta \leq 1/2$. When $\theta = 0$, there is no difference in terms of equilibrium outcomes between sellers forming passive beliefs or wary beliefs. When $\theta \leq 1/2$ (with $\theta \neq 0$), it holds that $w^* > \widetilde{w} = 0$, $p^* > \widetilde{p} > 0$ and $p_0^* < \widetilde{p}_0 < 0$, with $\pi_0^* < \widetilde{\pi}_0$ and $cs^* < \widetilde{cs}$.*

On the one hand, when sellers' beliefs are passive rather than wary, the platform's price structure is somewhat affected, with royalty fees falling down to zero and the markup earned on consumers rising but yet remaining below zero. Platform profits and consumer surplus are still below those attained were contracts public, which is consistent with our analysis in the previous sections.

On the other hand, platform profits and consumer surplus under passive beliefs are larger than under wary beliefs. As is well known from the vertical relations literature, passive beliefs imply a greater hold-up problem in dealing with sellers, which implies that seller prices decrease with respect to the case of wary beliefs. In a two-sided market, this loss of control in dealing with sellers serves as a commitment device for reducing the hold-up problem when dealing with consumers, and, as a result, the platform provider (and consumers) benefit when sellers form passive beliefs rather than wary.

Once again, this result shows how informational problems interact in a two-sided market, which has a significant effect on the optimal policies that should be implemented from a firm and social perspective. For example, Rey and Vergé (2004) show in the one-sided market case that, when $0 < \theta < 1/2$, an upstream monopolist earns less when downstream firms form passive beliefs than wary beliefs, which is just the opposite of what we find in the presence of two-sidedness.

Our analysis shows that the policy and managerial implications drawn in previous sections are even stronger when sellers form passive beliefs.

8. Sequential adoption decisions

In previous sections, we assumed that both sides of the market decide whether to join the platform at the same time. Most papers in the two-sided markets literature study this timing (see Rochet and Tirole, 2003; Armstrong, 2006, for example). In fact, one of the main concerns of the two-sided-markets literature is that the mutual dependency of platform-membership decisions may create a chicken-and-egg problem (Caillaud and Jullien, 2003), which is absent when groups of users make their membership decisions sequentially. Other papers assume that the members of one side (typically consumers) make their adoption decisions after observing the adoption decisions of the members of the other side (typically sellers). In particular, see Hagiu (2006). In this section, we study a model with this alternative timing and compare it with the model studied in Section 4.

Both timings are interesting from a practical point of view. Consider, for example, the decision to buy a video game console. When making their choices, gamers consider not only the games that have already been developed for that platform (which would be consistent with the timing of this section), but also the games that will be developed in the future (which is consistent with the timing of Section 4). The timing of Section 4 is likely to be more relevant when a new gaming platform is launched, but both timings are relevant for mature platforms with a large installed base of games.

We focus on the case in which the goods sold by sellers are independent (i.e., $\theta = 0$), and study a two-period model with the following timing.[21] In the first period, the platform sets p_0, which becomes public information, and the platform privately offers contract (f_i, w_i) to seller $i \in \{1, 2\}$, which chooses whether or not to accept the contract at the same time

[21] As will soon become clear, the model with sequential adoption decisions is highly complex, which implies that we have to resort to numerical solutions. For concreteness, we focus on the study of the model with $\theta = 0$, and leave the study of the cases with $\theta \neq 0$ for future research.

the other seller does. In the second period, sellers choose prices for their goods, and then consumers observe all prices and decide whether to join the platform as well as how many units to buy from each seller.

As in the previous sections, we assume that the platform includes p_0 into seller's contracts. Thus, there are no unobservability problems about price p_0 on the seller and the consumer sides. We also continue to assume that sellers have wary beliefs about the royalty fees offered to other sellers.

One would be tempted to speculate that Rey and Vergé's (2004) results extend trivially to this framework, since consumers are fully informed when making their decisions, and sellers' products are, from the point of view of consumer demands, independent. However, there are several differences between Rey and Vergé's (2004) model and this one. First, sellers' goods become complementary: when the price of one product rises, it decreases the number of consumers who join the platform, which has a negative effect on the total demand of the other product. Second, p_0 affects the interaction between sellers, so the platform may use p_0 as an instrument to modulate the effects of private information. Third, beliefs may now depend on p_0, because: (i) consumer demand depends on p_0, p_1 and p_2, so p_0 affects sellers' first-order conditions in the pricing subgames and; (ii) the platform's optimal deviation when it offers a contract to a seller depends on p_0 (which means that p_0 affects the formation of wary beliefs). These effects are absent in Rey and Vergé's (2004) paper.

We start by noting that the public information case of the model with the alternative timing is equivalent to the one studied in Section 3: if all prices are publicly observed, the platform provider behaves as an integrated firm, induces marginal cost pricing by sellers, and chooses a positive access fee for consumers. Note also that the intermediate case with informed sellers and uninformed buyers is equivalent to the public information case of Section 3, since consumers make their consumption decisions after observing p_i (thus, they do not need to form beliefs about royalty fees).

We turn now to the study of the private contracts case. When $\theta = 0$, we know that

$$q_i(p_i) = 1 - p_i \tag{11}$$

and

$$x_0(p_1, p_2, p_0) = \frac{(1-p_1)^2 + (1-p_2)^2}{2} - p_0. \tag{12}$$

Suppose that, upon observing w_i and p_0, seller $i \in \{1, 2\}$ follows pricing strategy $p_i(w_i, p_0)$ and believes that seller j follows pricing strategy $p_j(B(w_i, p_0), p_0)$ in the second period. Because

$$p_i(w_i, p_0) \in \arg\max_{p_i}\{(p_i - w_i)(1-p_i)x_0(p_i, p_j(B(w_i, p_0), p_0), p_0)\},$$

the first-order condition yields that

$$\Big(1 + w_i - 2p_i(w_i, p_0)\Big)\left(\frac{[1 - p_i(w_i, p_0)]^2 + [1 - p_j(B(w_i, p_0), p_0)]^2}{2} - p_0\right)$$
$$-[p_i(w_i, p_0) - w_i][1 - p_i(w_i, p_0)]^2 \;=\; 0.$$

Thus, by symmetry, in equilibrium the following functional equation must hold for all w and p_0:

$$\Big(1 + w - 2p(w, p_0)\Big)\left(\frac{[1 - p(w, p_0)]^2 + [1 - p(B(w, p_0), p_0)]^2}{2} - p_0\right)$$
$$-[p(w, p_0) - w][1 - p(w, p_0)]^2 \;=\; 0. \qquad (13)$$

To obtain the condition that corresponds to the first period, note that, upon observing w_2 and p_0, seller 2 believes that the platform chooses w_1 to maximize

$$x_0(p_1(w_1, p_0), p_2(w_2, p_0), p_0)\,[p_0 + w_1\, q_1(p_1(w_1, p_0)) + w_2\, q_2(p_2(w_2, p_0))] + f_1 + f_2$$

subject to

$$f_1 \leq (p_1(w_1, p_0) - w_1)\, x_0(p_1(w_1, p_0), p_2(B(w_1, p_0), p_0), p_0)\, q_1(p_1(w_1, p_0)),$$

so seller 2 believes that the platform chooses w_1 to maximize

$$x_0(p_1(w_1, p_0), p_2(w_2, p_0), p_0)[p_0 + w_1 q_1(p_1(w_1, p_0)) + w_2 q_2(p_2(w_2, p_0))] + f_2 +$$
$$[p_1(w_1, p_0) - w_1]x_0(p_1(w_1, p_0), p_2(B(w_1, p_0), p_0), p_0)q_1(p_1(w_1, p_0)). \qquad (14)$$

Working with the first-order condition we obtain the following differential equation (see Appendix C for details):

$$\begin{aligned}
0 \;=\; -\Bigg[& \Big(1 - p(B(w, p_0), p_0)\Big)\Big(p_0 + B(w, p_0)\,(1 - p(B(w, p_0), p_0)) + w\,(1 - p(w, p_0))\Big) \\
& + \big(p(B(w, p_0), p_0) - B(w, p_0)\big)\,\big(1 - p(B(w, p_0), p_0)\big) \\
& \times \Big(1 - p(B(B(w, p_0)), p_0)\Big)\,\frac{\partial B(B(w, p_0))}{\partial B(w, p_0)} \\
& + \left(\frac{(1 - p(B(w, p_0), p_0))^2 + (1 - p(w, p_0))^2}{2} - p_0\right) B(w, p_0)\Bigg]\,\frac{\partial p(B(w, p_0), p_0)}{\partial B(w, p_0)} \\
& + \left(\frac{(1 - p(w, p_0))^2 - (1 - p(B(B(w, p_0)), p_0))^2}{2}\right)\Big(1 - p(B(w, p_0), p_0)\Big).
\end{aligned} \qquad (15)$$

Given p_0, $p(w, p_0)$ and $B(w, p_0)$ solve the system of equations given by (13) and (15). The second-period equilibrium royalty fee, $w^a(p_0)$, solves $w^a = B(w^a, p_0)$. In the first period, the platform chooses p_0 to maximize profits, taking into account that the second-period equilibrium royalty fee is given by $w^a(p_0)$.

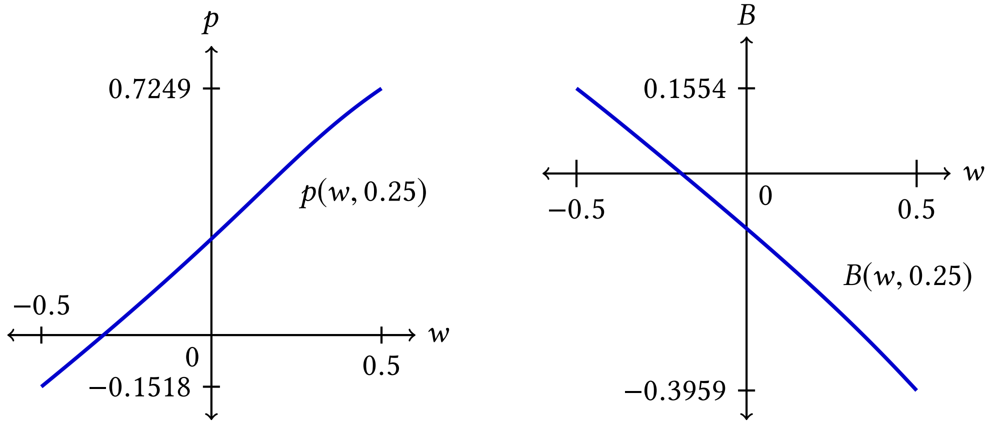

Figure 7. Price and belief functions for $p_0 = 0.25$

A general solution for a system of differential equations is not known. Our attempts to solve the system comprised of equations (13) and (15) were unsuccessful. Thus, we proceeded to obtain a numerical solution.

We develop an algorithm using a combination of Euler's method and the fourth-order Runge-Kutta method.[22] For a given p_0, the algorithm yields a numerical approximation of $p(w, p_0)$ and $B(w, p_0)$. Figure 7 shows $p(w, p_0)$ and $B(w, p_0)$ for $p_0 = 0.25$. We apply the algorithm for different values of p_0 and find the one that maximizes the platform's profit. Figure 8 shows first-period profits for the platform for different values of p_0. The optimal platform price is $p_0^a = 0.4785$, the equilibrium royalty fee is $w^a = -0.1582$, the equilibrium price for sellers is $p^a = 0.1012$, equilibrium platform profits are $\pi^a = 0.2172$, and consumer surplus is $cs^a = 0.0539$.[23]

In comparison, in the public-contract case (Section 3), $p_0^t = 0.5$, $w^t = -1$, $p^t = 0$, $\pi_0^t = 0.25$, and $cs^t = 0.125$; and in the private-contract case with the original timing of Section 4.2, $p_0^* = -0.125$, $w^* = 0$, $p^* = 0.5$, $\pi_0^* = 0.1406$, and $cs^* = 0.0703$ (when $\theta = 0$, the results of the intermediate case of Section 4.1 coincide with the private-contract case).

The analysis of private contracts with sequential adoption shows two main results. First, as in the model with the original timing of Sections 3 and 4, equilibrium profits and consumer surplus are smaller with private contracts than in the public information case, but the price structure with the alternative timing is closer to the price structure of the public-contract case. Second, platform profits are larger and consumer surplus is smaller than in the case of private contracts with the original timing. Therefore, private contracts have a

[22]See Butcher (2016) for an introduction to numerical methods for differential equations. Details of the algorithm are available upon request.

[23]Given the numerical nature of our solution, these results are approximate. Most importantly, they depend on our choice of: (i) the increment used to calculate numerical derivatives, (ii) the functional form of the approximation error, and (iii) the algorithm's stopping rule. We have conducted sensibility tests which show that the numerical results are robust to changes in these parameters. Results are available upon request.

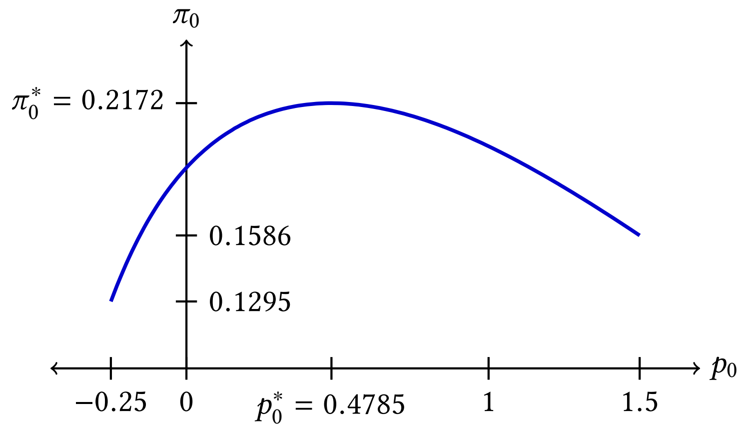

FIGURE 8. Platform profits in the continuation game

more negative effect on consumers with the alternative timing than with the original timing. These results show that informational problems are important even when consumers make their adoption decisions after sellers. While these results are suggestive, understanding the effects of private contracts when consumers and sellers make adoption decisions at different times requires solving a fully dynamic model, which we leave for further research.

9. Concluding remarks

When contracts between the platform and sellers are private rather than public, we have shown that the pricing structure is basically driven by consumers' fear of being taken advantage of when purchasing the platform. Transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers. Another way to make adverse selection disappear is to contract on sellers' prices (as in the case of iTunes). Both the platform and consumers would benefit from the use of such price-forcing contracts. Also, we have shown that integration of the platform provider with both sellers can allow it to return to the public contracting outcome and do better without harming consumers.

Our results show that giving more market power to a dominant platform (e.g., in the form of allowing for price-forcing contracts) may make everybody better off because it removes information frictions: everyone benefits from consumers not being so pesimistic about the value delivered by the platform. This insight does not only apply when consumers purchase the platform without observing the costs associated to using it, but rather it is more general. It also holds in cases in which consumers do not observe the quality of the goods sold by sellers before acquiring the platform, or when they do not observe the full variety of goods that will be offered through the platform. This may explain why quality assurance by platforms is common (as is the case for Nintendo). We believe that these topics present an interesting direction for further research.

Appendix A. Proof of Proposition 2

If $p(w) = \Theta + \Sigma w$ and $B(w) = \Gamma + \Phi w$ for some parameters Θ, Σ, Γ and Φ to be determined, conditions (7) and (10) can be rewritten as

$$(1-\theta)(1-\Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) + [2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1)]w_2 = 0$$

and

$$1 - \theta + \theta\Sigma\Gamma - (2-\theta)\Theta + (1 - 2\Sigma + \theta\Sigma\Phi)w_2 = 0.$$

Since these two conditions should be satisfied for all w_2, we must have

$$(1-\theta)(1-\Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) = 0, \tag{16}$$

$$2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1) = 0, \tag{17}$$

$$1 - \theta + \theta\Sigma\Gamma - (2-\theta)\Theta = 0 \tag{18}$$

and

$$1 - 2\Sigma + \theta\Sigma\Phi = 0. \tag{19}$$

Rey and Vergé (2004) have already shown that there exists a unique tuple $(\Theta, \Sigma, \Gamma, \Phi)$ that solves these equations and the required second-order conditions for the platform's maximization program, but we will give closed-form solutions that will prove useful later on.

When $\theta = 0$, it is easy to see that there is a unique solution to equations (16)-(19), given by $\Theta = 1/2$, $\Sigma = 1/2$, $\Gamma = 0$ and $\Phi = 0$. From (19), one obtains

$$\Phi = \frac{2\Sigma - 1}{\theta\Sigma},$$

since it can be shown that there can be no solution with $\Sigma = 0$. Plugging this value for Φ in (17) allows us to rewrite it as the following cubic equation:

$$\Sigma^3 - \left(\frac{7-\theta^2}{2}\right)\Sigma^2 + \frac{5}{2}\Sigma - \frac{1}{2} = 0. \tag{20}$$

Letting

$$a \equiv -\frac{7-\theta^2}{2},$$

$$b \equiv \frac{5}{2},$$

$$c \equiv -\frac{1}{2},$$

$$K \equiv \frac{3b - a^2}{9}$$

and

$$L \equiv \frac{9ab - 27c - 2a^3}{54},$$

the solutions to the cubic equation are the following:

$$\Sigma_k = 2\sqrt{-K}\cos\left(\frac{1}{3}\arccos\left(\frac{L}{\sqrt{-K^3}}\right)+\frac{2\pi k}{3}\right)-\frac{a}{3}\ (k=0,1,2).$$

The three roots are real, given that the discriminant K^3+L^2 is negative for all $\theta \in (-1,1)$. Plotting the three roots for all values of θ, it is easy to see that the only one which is equal to $1/2$ when $\theta = 0$ is Σ_2. Given that the solution must be continuous in θ, we know that $\Sigma = \Sigma_2$, that is,

$$\Sigma = \frac{7-\theta^2}{6} - \frac{(19-14\theta^2+\theta^4)^{1/2}}{3}\sin\left(\frac{\pi}{6}-\frac{1}{3}\arccos\left(\frac{(1-\theta^2)(82-20\theta^2+\theta^4)}{(19-14\theta^2+\theta^4)^{3/2}}\right)\right).$$

From equation (18), we obtain

$$\Gamma = \frac{(2-\theta)\Theta-(1-\theta)}{\theta\Sigma},$$

so plugging it into (16) and rearranging yields that

$$\Theta = \frac{(1-\theta)[(6+\theta)\Sigma-2(1+\Sigma^2)]}{4(3-\Sigma)\Sigma+2\theta-(3\theta+\theta^2)\Sigma-4}.$$

It therefore follows from (16) that

$$\Gamma = \frac{(1-\theta)(2\Sigma-1)}{4(3-\Sigma)\Sigma+2\theta-(3\theta+\theta^2)\Sigma-4}.$$

Making it explicit that Θ, Σ, Γ and Φ depend on θ by writing Θ_θ, Σ_θ, Γ_θ and Φ_θ, it is easy to plot them and see that $0 \le \Theta_\theta \le 1$, $1/2 \le \Sigma_\theta \le 1$, $0 \le \Gamma_\theta \le 1$ and $-1 \le \Phi_\theta \le 1$ for all $\theta \in (-1,1)$. Note that beliefs must be fulfilled in equilibrium, so $w^* = B(w^*)$ implies that

$$w^* = \frac{\Gamma_\theta}{1-\Phi_\theta} \ge 0.$$

Also, the platform should find it optimal to choose $p_0 = p_0^*$ and $w_1 = w_2 = w^*$, so $(w^*, w^*, p_0^*) \in \underset{w_1,w_2,p_0}{\operatorname{argmax}}\ \widehat{\pi}_0(w_1,w_2,p_0)$, where

$$\begin{aligned}\widehat{\pi}_0(w_1,w_2,p_0) &= x_0(p_0)\big\{p_0 + w_1\, q_1(p_1(w_1),p_2(w_2)) + w_2\, q_2(p_2(w_2),p_1(w_1)) \\ &\quad +[p_1(w_1)-w_1]\, q_1(p_1(w_1),p_2(B(w_1))) \\ &\quad +[p_2(w_2)-w_2]\, q_2(p_2(w_2),p_1(B(w_2)))\big\}.\end{aligned}$$

Note that the optimal choices of w_1 and w_2 do not depend on the choice of p_0, so the platform provider can maximize with respect to w_1 and w_2 ignoring the value of p_0; the analysis above leading to expression (10) shows that private offers are chosen optimally, since second-order conditions are satisfied. To see this, note that (9) and the fact that

$$\frac{dq_1(p_1(w_1),p_2(B(w_1)))}{dw_1} = \frac{1}{1-\theta^2}\left(\frac{dp_1(w_1)}{dw_1}-1\right),$$

imply that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} = \frac{2(\Sigma_\theta^2 - 3\Sigma_\theta + 1)}{1 - \theta^2}$$

and

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} = \frac{2\theta\Sigma_\theta}{1 - \theta^2}.$$

Thus, it follows from the fact that $\Sigma_\theta \geq 1/2$ that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \leq 0.$$

Also, it holds that

$$\left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2}\right)^2 - \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2}\right)^2$$
$$= \frac{\Sigma_\theta(\Sigma_\theta^2 - 3\Sigma_\theta + 1)(\Sigma_\theta - 1) - (2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) - \theta^2\Sigma_\theta^2}{\left(\frac{1-\theta^2}{2}\right)^2},$$

which is nonnegative because $1/2 \leq \Sigma_\theta \leq 1$ and $(2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) + \theta^2\Sigma_\theta^2 = 0$ by (20). Thus, second-order conditions hold.

As for the optimal choice of p_0 given that seller $i \in \{1, 2\}$ receives an offer equal to (w^*, f^*), we need that

$$x_0(p_0) + [p_0 + 2p_1(w^*)q_1(p_1(w^*), p_2(w^*))]\frac{dx_0(p_0)}{dp_0} = 0,$$

so

$$p_0^* = \frac{(1 - w^*)^2}{2(1 + \theta)(2 - \theta)^2} - \frac{2(\Theta_\theta + \Sigma_\theta w^*)(1 - \Theta_\theta - \Sigma_\theta w^*)}{2(1 + \theta)},$$

which is negative for all $\theta \in (-1, 1)$. Finally, note that

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta\Gamma_\theta}{1 - \Phi_\theta} \; (i \in \{1, 2\}),$$

so $0 \leq p_i^* \leq 1$. It readily follows that

$$q_i^* = \frac{1 - p_i^*}{1 + \theta} > 0,$$

$$\pi_0^* = \left(\frac{1 - p_i^{*2}}{2(1 + \theta)}\right)^2,$$

and

$$cs^* = \frac{1}{2}\left(\frac{1 - p_i^{*2}}{2(1 + \theta)}\right)^2.$$

Appendix B. Proof of Proposition 8

Since $\widetilde{w} = 0$, it holds that $w^* \geq \widetilde{w}$, with equality if and only if $\theta = 0$. Figure 9 shows that the price charged by sellers when they form passive beliefs is lower than that when beliefs are wary (the dashed curve corresponds to the case of wary beliefs).

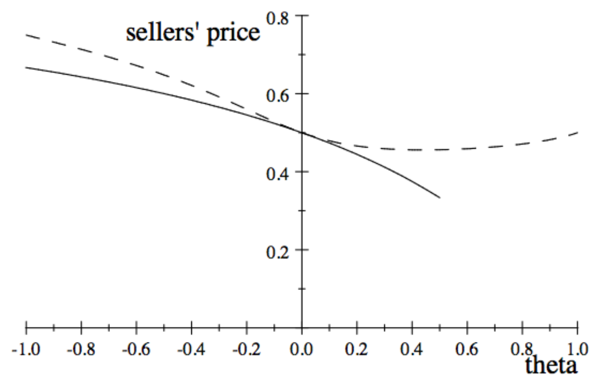

Figure 9. Comparison of seller prices

Figure 10 shows that the platform's price under passive beliefs is greater than that under wary beliefs (the dashed curve corresponds to the case of wary beliefs).

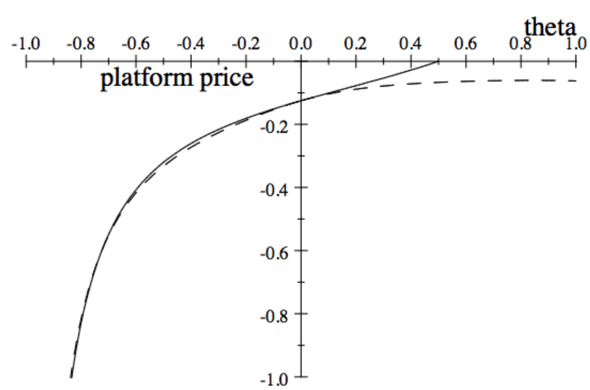

Figure 10. Comparison of platform's access price

Finally, profits under passive beliefs equal

$$\widetilde{\pi}_0 = \Pi(\widetilde{w}, \widetilde{p}_0) = \frac{(3 - 2\theta)^2}{4(1 + \theta)^2(2 - \theta)^4}.$$

These profits exceed those under wary beliefs, as illustrated by Figure 11 (the dashed curve corresponds to the case of wary beliefs).

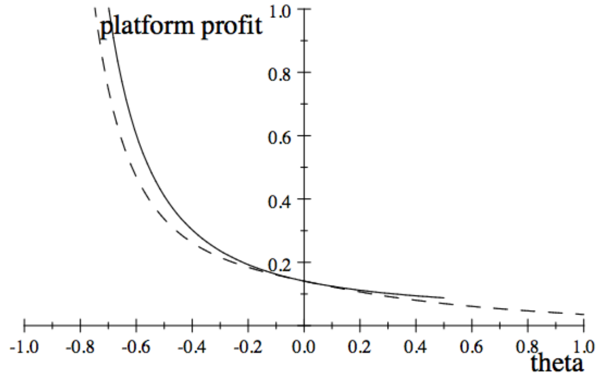

FIGURE 11. Comparison of platform profit

Consumers' welfare is half of the platform's profit when beliefs are wary, and the same applies to passive beliefs, since

$$\begin{aligned}\widetilde{cs} &= \int_0^{x_0} (\frac{(1-c-\widetilde{w})^2}{(1+\theta)(2-\theta)^2} - \widetilde{p}_0 - x)dx \\ &= \frac{(3-2\theta)^2}{8(1+\theta)^2(2-\theta)^4}.\end{aligned}$$

APPENDIX C. PROCEDURE FOR OBTAINING EQUATION (15)

Introducing (11) and (12) into (14), we obtain

$$\left(\frac{(1-p_1(w_1,p_0))^2 + (1-p_2(w_2,p_0))^2}{2} - p_0\right)\left(p_0 + w_1(1-p_1(w_1,p_0)) + w_2(1-p_2(w_2,p_0))\right) + f_2 +$$
$$\left(\frac{(1-p_1(w_1,p_0))^2 + (1-p_2(B(w_1,p_0),p_0))^2}{2} - p_0\right)\left(p_1(w_1,p_0) - w_1\right)\left(1-p_1(w_1,p_0)\right).$$

The first-order condition with respect to w_1 is

$$\begin{aligned} 0 = & -\left(1-p_1(w_1,p_0)\right)\frac{\partial p_1(w_1,p_0)}{\partial w_1}\left(p_0 + w_1\left(1-p_1(w_1,p_0)\right) + w_2\left(1-p_2(w_2,p_0)\right)\right) \\ & + \left(\frac{(1-p_1(w_1,p_0))^2 + (1-p_2(w_2,p_0))^2}{2} - p_0\right)\left(1-p_1(w_1,p_0) - w_1\frac{\partial p_1(w_1,p_0)}{\partial w_1}\right) \\ & - \left(\left(1-p_1(w_1,p_0)\right)\frac{\partial p_1(w_1,p_0)}{\partial w_1} + \left(1-p_2(B(w_1,p_0),p_0)\right)\frac{\partial p_2(B(w_1,p_0),p_0)}{\partial B(w_1,p_0)}\frac{\partial B(w_1,p_0)}{\partial w_1}\right) \\ & \times \left(p_1(w_1,p_0) - w_1\right)\left(1-p_1(w_1,p_0)\right) \\ & + \left(\frac{(1-p_1(w_1,p_0))^2 + (1-p_2(B(w_1,p_0),p_0))^2}{2} - p_0\right) \\ & \times \left(\left(\frac{\partial p_1(w_1,p_0)}{\partial w_1} - 1\right)\left(1-p_1(w_1,p_0)\right) - \left(p_1(w_1,p_0) - w_1\right)\frac{\partial p_1(w_1,p_0)}{\partial w_1}\right). \end{aligned}$$

Replacing w_2 by w, and w_1 by $B(w_2, p_0)$, and noting that $p_1(\cdot)$ and $p_2(\cdot)$ should be equal because of symmetry, we obtain

$$
\begin{aligned}
0 \;=\; & -\Big(1 - p(B(w,p_0),p_0)\Big)\,\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)} \\
& \times \Big(p_0 + B(w,p_0)\,(1 - p(B(w,p_0),p_0)) + w\,(1 - p(w,p_0))\Big) \\
& + \left(\frac{(1 - p(B(w,p_0),p_0))^2 + (1 - p(w,p_0))^2}{2} - p_0\right) \\
& \times \left(1 - p(B(w,p_0),p_0) - B(w,p_0)\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)}\right) \\
& - \Bigg[\Big(1 - p(B(w,p_0),p_0)\Big)\,\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)} \\
& + \Big(1 - p(B(B(w,p_0)),p_0)\Big)\,\frac{\partial p(B(B(w,p_0)),p_0)}{\partial B(B(w,p_0))}\,\frac{\partial B(B(w,p_0))}{\partial B(w,p_0)}\Bigg] \\
& \times \big(p(B(w,p_0),p_0) - B(w,p_0)\big)\,\big(1 - p(B(w,p_0),p_0)\big) \\
& + \left(\frac{(1 - p(B(w,p_0),p_0))^2 + (1 - p(B(B(w,p_0)),p_0))^2}{2} - p_0\right) \\
& \times \left(\left(\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)} - 1\right)\,\Big(1 - p(B(w,p_0),p_0)\Big)\right. \\
& \left. - \Big(p(B(w,p_0),p_0) - B(w,p_0)\Big)\,\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)}\right).
\end{aligned}
$$

Together with (13), this differential equation, that should hold for all w and p_0, gives $p(w, p_0)$ and $B(w, p_0)$. Note that (13) implies that

$$
\begin{aligned}
& \left(\frac{(1 - p(B(w,p_0),p_0))^2 + (1 - p(B(B(w,p_0)),p_0))^2}{2} - p_0\right) \\
& \times \left(\Big(1 - p(B(w,p_0),p_0)\Big) - \Big(p(B(w,p_0),p_0) - B(w,p_0)\Big)\right) \\
& \quad = \Big(1 - p(B(w,p_0),p_0)\Big)^2\,\big(p(B(w,p_0),p_0) - B(w,p_0)\big).
\end{aligned}
$$

Thus, the above expression simplifies to

$$\begin{aligned}
0 \quad = \quad & -\Big(1 - p(B(w,p_0),p_0)\Big)\,\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)} \\
& \times \Big(p_0 + B(w,p_0)\,(1 - p(B(w,p_0),p_0)) + w\,(1 - p(w,p_0))\Big) \\
& + \left(\frac{(1 - p(B(w,p_0),p_0))^2 + (1 - p(w,p_0))^2}{2} - p_0\right) \\
& \times \left(1 - p(B(w,p_0),p_0) - B(w,p_0)\frac{\partial p(B(w,p_0),p_0)}{\partial B(w,p_0)}\right) \\
& - \left(\Big(1 - p(B(B(w,p_0)),p_0)\Big)\,\frac{\partial p(B(B(w,p_0)),p_0)}{\partial B(B(w,p_0))}\frac{\partial B(B(w,p_0))}{\partial B(w,p_0)}\right) \\
& \times \big(p(B(w,p_0),p_0) - B(w,p_0)\big)\,\big(1 - p(B(w,p_0),p_0)\big) \\
& - \left(\frac{(1 - p(B(w,p_0),p_0))^2 + (1 - p(B(B(w,p_0)),p_0))^2}{2} - p_0\right) \\
& \times \Big(1 - p(B(w,p_0),p_0)\Big),
\end{aligned}$$

which is equivalent to equation (15).

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